VOLUME XLVII NUMBER TWO

The Mathematics Teacher

FEBRUARY 1954

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The official journal of
THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

The Mathematics Teacher is a journal of The National Council of Teachers of Mathematics devoted to the interests of mathematics teachers in the Junior High Schools, Senior High Schools, Junior Colleges and Teacher Education Colleges.

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Printed at Menasha, Wisconsin, U.S.A. Entered as second-class matter at the post office at Menasha, Wisconsin. Acceptance for mailing at special rate of postage provided for in the Act of February 28, 1925, embodied in paragraph 4, section 412 P. L. & R., authorized March 1, 1930. Printed in U.S.A.

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A new responsibility of teacher education programs

LEE EMERSON BOYER *of Millersville State Teachers College, Millersville, Pennsylvania,* *raises an important and puzzling question concerning the teacher education programs and their adequacy for preparing teachers for the functional or "basic" courses in mathematics. He realizes that his proposals are not likely to receive universal approval. Houston Karnes spells out some objections to Mr. Boyer's proposals in the "Points and Viewpoints" columns of this issue.*

DURING THE PAST DECADE widespread revision of mathematics courses of study on the state level has taken place. One outstanding characteristic of these revisions, taken as a whole, is that in their attempt to correct educational ills of many years' standing they suggest flexible mathematics programs for all high school pupils throughout their stay in high school. These programs are varied and planned to meet the needs of various types of pupils. Although there can really be a great number of mathematics programs offered in a large high school, especially if "sectioning" is used to its full advantage, this paper will simply divide the mathematics offering into two main programs: the older traditional, sequential, or *specialized* mathematics program; and the new general, functional, or *basic* mathematics program.[1] The new responsibility referred to in the title of this paper is: How are teacher-training institutions going to help get the new state curriculums into successful operation? More pointedly, the challenge may be stated: What are teacher-training institutions going to do to make the successful operation of the basic mathematics program more probable?

This challenge is a very important one. It seems to the writer that, during the next quarter of a century, under the provisions of the new courses of study, mathematics supervisors and teachers will have a great opportunity to bring the study of mathematics in the American secondary school to its proper place. Like every other opportunity this one must be studied relentlessly and developed with extreme care and sound judgment lest the good that is hoped for will not materialize.

In the beginning of our thinking, let us bear in mind how it happened that the study of mathematics became an elective some years ago. It wasn't that mathematics as a subject was not regarded as one ideally fitted for school use. With respect to its crystallized subject matter, the ease and economy with which the subject matter can be presented to relatively large groups of pupils, the intellectual challenge which the subject matter offers,

[1] The new *Study Guide for Mathematics in Pennsylvania* uses the adjectives "specialized" and "basic"; California uses "college preparatory" and "basic"; Florida, "traditional" and "functional."

and the possibilities for significant applications of both the content and the methods of the subject have never been seriously questioned. Yet mathematics became an elective, elected by proportionately fewer and fewer pupils, when the compulsory attendance laws brought practically all children of high school age into school. But it wasn't mere numbers of students that caused the change in curriculum status. Frankly, mathematics became an elective because it was impossible for a sufficiently large proportion of these run-of-the-mine pupils to pass the mathematics subjects as they were then organized and taught. In other words, there were so many "failures" in mathematics that educators were forced, by rather sound administrative principles, to declare mathematics an elective. For us, interested in mathematics and especially interested in mathematics for all pupils, it is particularly important that we comprehend the extent to which the study of mathematics has suffered because of a certain unwillingness on the part of mathematicians to reorganize the content and to teach the subject in such a way that its bearing on the problem of living would be more obvious.

If, within the near future, our schools are to offer successfully the basic mathematics recommended in the new curriculums, the teacher-training institutions must immediately think about what they can do with respect to these three questions:

1. How may we produce the additional mathematics teachers that will be needed?
2. What type of course instruction shall we give the new (not in-service) prospective teachers of basic mathematics?
3. What methods of teaching ought the new prospective teachers of basic mathematics experience?

It is obvious that, if every high school pupil in our schools should want to enroll in a mathematics course next year, we would not have nearly enough mathematics teachers to staff the classes; we might have about half enough. All of us who have been in the teacher preparation business for some years know too well the makeshift teacher assignments that are likely to be made when teachers are scarce. It is to guard against having just any teacher at all teach a very special kind of mathematics at a crucial period in our mathematics program that we are here raising our voices. In our efforts to secure the necessary number of mathematics teachers, we should, of course, employ all of the more or less standardized procedures of good teacher recruitment. These procedures need not be repeated here except to say that by and large teacher-educating institutions have not yet put into regular and systematic practice the theories of recruitment they know to be good. It behooves us, therefore, to support the speedy initiation and development of these programs. In addition, however, we must emphasize to mathematics teachers, other subject teachers, parents, school administrators, and the public in general the very special problem in mathematics education that confronts us at this moment.

Teaching the new basic mathematics courses will be a really difficult task; a task which should be undertaken only by teachers who are thoroughly acquainted with the objectives of the basic mathematics program and who are willing to give their best efforts to make the course one of the most profitable of any offered in the school. In the opinion of the writer, it would be wise for a school to expand its mathematics program only after the proper teaching personnel is available; for here, as almost nowhere else, the success of the course will depend upon the training and the attitude of the teacher.

Only with great care should mathematics teachers be placed in the two mathematics areas and in the various sections. Fortunately, there are some teachers now teaching mathematics that can handle the new basic mathematics. It

might be wise to give these experienced teachers who are in sympathy with this mathematics program the opportunity to take as much of the work in basic mathematics as they can. With their teaching experience and with a little experimentation they would be the best teachers for the new job. Teacher-training institutions should help these teachers by offering them seminars, work-shops, part-time courses, and conferences dealing with basic mathematics. Millersville offered such a conference on March 10. In addition to a lecture on "Implementing a Philosophy of Basic Mathematics," four teaching demonstrations with basic mathematics materials were conducted. These demonstrations allowed teachers to see subject matter presented to pupils in ways regarded as helpful in accomplishing the objectives of basic mathematics. Discussion of the procedure followed with special efforts to induce listening teachers to share their related ideas and experiences. Lists of related textbooks, pamphlets, information sheets, study guides, and magazine articles that are available can also be helpful.

It seems reasonable that, no matter what we can do with the basic mathematics program, the quality of the specialized mathematics program will be improved by offering basic mathematics. That this seems possible is due to the fact that now, more readily than ever before, both teachers and pupils can be grouped for a single teaching purpose. In the specialized program, the students can probably be taught best by teaching the *subject* primarily. The traditional study of mathematics from a logical, perhaps research, point of view seems highly promising. For this task our present machinery for preparing mathematics teachers seems to be fairly well suited. The traditional mathematics courses, the textbooks now available, and the teaching staffs in our colleges were all, in the main, prepared for just such a job. This task of preparing teachers to teach the specialized mathematics to high school pupils who are interested in areas demanding for the most part the use of mathematics as a tool should be strengthened rather than weakened. Teacher-training students in this program should study throughout their college career courses rich in rigor and abstractions. The ground covered will be extensive and the pace rapid. The demonstrations will often be brief, and frequently the lecture method of teaching will be used to advantage.

Such a college program, however, would not be a good one for the prospective teacher of basic mathematics; for many of the habits and points of view established by such study would handicap the basic mathematics teacher whose main emphasis in his teaching will be to show the pupil how the study of mathematics can help him and (later) his pupils better solve the problems of life. Here the *student* with his need to interpret life problems, rather than the subject, becomes the focal point of teaching. There will be, to be sure, some overlapping of objectives in the specialized and basic courses. In Pennsylvania, we speak of these as "the common learnings." But, in the main, there will be a marked difference. The courses for prospective basic mathematics teachers should emphasize applications, practical problems, short-cut rules of operation, maintenance of arithmetic skills, and utilitarian aspects of geometry, algebra and trigonometry. The "ground to be covered" from the mathematics subject-matter point of view will be less extensive; more attention will be given to correspondences to life situations. The pace and depth of probing will be governed by the pupils' needs rather than by the demands of any later study of a particular body of subject matter. Much more time and effort than in the specialized courses will be given to exemplifying how the laws of learning and motivation operate in learning mathematics. Field work instruments, models, films, and other visual aids will be used much more extensively. The primary aim will be to

promote the all-round development of the student through a study of mathematics.

In view of the foregoing and the very significant fact that young teachers tend to teach as they were taught, teacher-training institutions, it would seem, should set up two "tracks" for training mathematics teachers to correspond with the "tracks" in high school. Some will be inclined to say that we should include some of the necessary methodology in our already fairly well established training program for prospective teachers of specialized mathematics, and in that way accommodate the prospective teachers of basic mathematics. This, however, is an easy but highly make-shift procedure. Since we actually need many more basic mathematics teachers than we need teachers of specialized mathematics, and since the basic mathematics program is new at this time, would it not be much more reasonable, if we must have only one mathematics teacher-training program, to plan it for the vast majority of students—the prospective basic mathematics teachers—and let the prospective specialized mathematics teachers be accommodated by certain convenient modifications? An affirmative answer to this question would work so mightily against the proper training of our scientists of the future that it is certain that the answer must be an emphatic "No more finagling of programs that work harm against the gifted pupils." The only tenable position, then, is a two-track program for prospective mathematics teachers.

Just what form the "track" for preparing basic mathematics teachers should take is a problem beyond the scope of this paper. The point to be made here is that teacher-training institutions sense the size and reality of the problem and make direct efforts to meet it successfully. Our steps at Millersville along this line include a course, Field Work in Mathematics, emphasis on construction and use of teaching models, numerous supplementary materials rich in applications of mathematics, special attention to certain helpful issues concerned with individual differences in the courses Teaching of Secondary Mathematics and History of Mathematics, and an attempt to get students to study their own teaching powers and equipment so they will be especially well prepared either for the job of teaching the specialized mathematics or for that of teaching basic mathematics.

The following can be said with respect to methods of teaching that shall be used with prospective teachers of basic mathematics: (1) the learning must be made interesting; and (2) the learning must be accomplished more through basic experiences than through assign-study-recite techniques. In the past, the cause of general mathematics suffered greatly because too often it was regarded by administrators, teachers, counsellors, and pupils as a course for "dull," "slow," "non-gifted," or "inferior" pupils who surely would "never amount to anything anyway." Under such conditions one could scarcely expect much interest, enthusiasm, or growth. With our present-day understanding of individual differences among people, intellectually, vocationally, and in many other ways, and with our present-day assumption that high schools are supported to serve all pupils of high-school age, who can maintain that one set of courses is more important than another? In the eyes of society, one individual is just as important as any other. Therefore, every high school administrator, counsellor, and teacher is duty-bound to give his full support to making all courses attractive to and helpful for the pupils for whom they are designed. It does not follow at all that the resulting courses will be "soft" or "watered down" or wasteful. It does follow that they will be challenging to the people taking the course, and that, by and large, pupils in them will have a reasonable opportunity to experience success.

The methods of teaching basic mathematics should exemplify life investigations. Formulating and solving problems,

gathering, treating and interpreting data, approximations, functions, operations, proofs, and use of symbols in the solution of actual problems will, in themselves, go a long way toward suggesting proper methods because acceptable results will necessitate understanding the elements involved. In this kind of teaching, the personality of the learner should be given much attention. Inter-personal and group relationships should be relied upon to furnish experiences needed to learn by doing. The atmosphere of the classroom should change from that of a lesson-hearing room to that of a learning laboratory. As such, the number of pupils in any one class should not exceed thirty. There should be no hesitation to use mathematics-class time to clarify mathematical issues in other subject classes. An over-all guide with respect to methods of teaching basic mathematics classes might be: It is well to promote sound judgment and to do good. That such a maxim for mathematics teaching may seem strange, the writer admits. But is it stranger than to contemplate teaching mathematics successfully to all secondary-school pupils? That's a big order. There seems to be no surer approach than one based on a sincere search for understanding and human values. That the study of mathematics is especially helpful in these respects has been held by many. Our own Ben Franklin, an educator of no small dimensions, asked years ago, "What science can there be more noble, more excellent, more useful for men, more admirably right and demonstrative, than this of mathematics?"

Have you read?

CAIRNS, S. S. "Elementary and Secondary School Training in Mathematics," *The American Mathematical Monthly*, October 1953, pp. 523–527.

"Our schools are sadly deficient both in preparing students for college and in offering adequate education to those not bound for college. Our elementary schools, in turn, are deficient in preparing students for high school." This is a statement made by Mr. Cairns. He substantiates his statement with research data. For example, would you estimate that approximately 50% of 500 college students failed to correctly combine $7-6+2-4$, even though these students had completed two years of high school mathematics. The author points out some causes of this and then proposes a guiding principle to be followed in the future so that such a situation will no longer exist. In spite of the negative approach and broad generalizations which are made there is much that we mathematics teachers must seriously consider in this article.

EINAR, HILLE. "Mathematics and Mathematicians from Abel to Zermelo," *Mathematics Magazine*, January–February 1953, pp. 127–146.

Every 100 years we should restudy our development because through it we note the trends we will probably follow in the future. The article approaches the history of mathematics through "Who was who in Mathematics in 1852." It also gives the development of the complex function theory over a period of 100 years. It provides good reading for high school students in that it brings before them the men whose work provided the impetus for the 20th century growth and our position in mathematics. Such names as Cauchy, Louisville, Sturm, Gauss, Jacobi, Abel, Steiner, Mobius, Rieman, Dedekind, Green, Stokes, Cayley, Sylvester, Boole, Morgan, Bolyai, Weierstrauss and others are mentioned. Although the article is brief it provides a connected dissertation on each man's contribution and its value to the field of mathematics.

KNOBLOCH, IRVING WILLIAM, "The Teacher's Creed," *School Science and Mathematics*, October 1953, pp. 513–515.

What should a teacher live by? Mr. Knobloch has set forth seven statements to be used as a guide in the everyday dealings of the teacher. They cover not only the specifics in the classroom but extend to the community and the world. He also gives a nice explanation of each. You may disagree with his creed, but you must agree that because the teacher holds a key position he needs to be concerned about the path he travels and leads his followers.—*Philip Peak, Indiana University, Bloomington, Indiana.*

The angle mirror—a teaching device for plane geometry

LAUREN G. WOODBY, *State Teachers College, Mankato, Minnesota.*

The uses of a simple angle mirror are brought to the attention of teachers of plane geometry. Junior high school teachers might well consider the feasibility of using the angle mirror as an interest-catching device.

THE ANGLE MIRROR, historically the forerunner of the sextant, is an inexpensive but interesting instrument, well suited for use by the teacher of geometry. The name "optical square" indicates the principal use of the angle mirror, namely to lay out a right angle.

Several commercial models are available, but student-made angle mirrors are perhaps even more desirable. The instrument consists simply of two mirrors, each perpendicular to the same plane, usually forming a dihedral angle of 45°.[1] An observer, looking in the direction of A (Fig. 1), lines up object B as seen by the double reflection, and thus the angle mirror becomes the vertex of the right angle AMB. Proof of the angle mirror is recommended as an exercise. The generalized proof of course holds for the sextant, which is essentially an angle mirror with a device for measuring the dihedral angle of the mirrors.

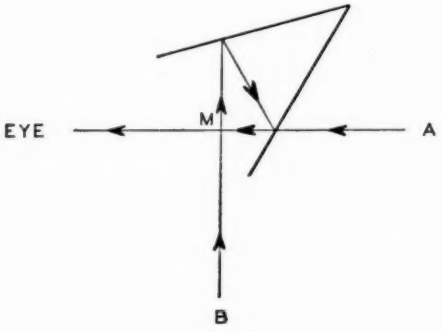


Figure 1

While the mirrors can be set approximately at 45° by the use of a draftsman's triangle or an angle formed by paper folding, there is a real challenge to the student in the field exercise of adjusting and checking the instrument. The fact that the mirrors can never be set exactly is disturbing to most students. One outcome of this exercise is the realization that any instrument, regardless of its precision, requires adjustment. Errors can be minimized but never completely eliminated. Shuster and Bedford present several field-tested methods of checking the angle mirror.[2]

The angle mirror is especially suited to give meaning to the concept of locus. For example, a pupil holding an angle mirror can walk so that two fixed points remain in alignment, thus illustrating the locus of the vertex of the right angle in a triangle whose hypotenuse is fixed. Striking results

[1] See "Devices for a Mathematics Laboratory," THE MATHEMATICS TEACHER (Oct. 1950), p. 287, for instructions for making a 45° angle mirror.

[2] C. N. Shuster and F. L. Bedford, *Field Work in Mathematics* (East Palestine, Ohio: Yoder Instrument Company).

are obtained when this work is done on the playground in freshly fallen snow. Another technique is to have several people occupy the various positions of the vertex of the right angle. Both these applications can be generalized using any setting of the mirrors.

The use of an angle mirror to locate the foot of an altitude in a triangle is a rich experience for the gifted youngster as well as for the slow learner. Practical work in finding the area of an irregular plot of ground gives meaning to the principles and concepts involved.

Use of the angle mirror is not restricted to the out-of-doors. It is pointed out that some "field work" can be carried out very effectively in the classroom. For example, the height of the ceiling can be determined indirectly, using the angle mirror and a tape. The tape is used to measure the height of the mirror and the distance from the wall.

The writer found an excellent use of the angle mirror in the work applying the Pythagorean Theorem. For the hypotenuse, a 100′ line is laid out to a precision of the nearest tenth of a foot ($100.0 \pm .05$). A point is then established for the vertex C of the right angle, using the angle mirror (Fig. 2). Next the foot of the altitude on

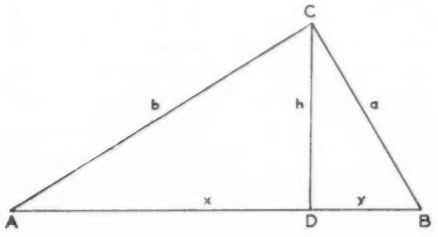


Figure 2

the hypotenuse is found by a student who walks from B toward A until C becomes aligned with A in the mirror. Various instruments such as a foot rule, a yardstick, a cloth tape, a carpenter's rule, or a steel tape are used to determine the lengths of h, x, and y. These distances, measured by several different pairs of individuals, result in a set of data that can be taken back to the classroom. A tabulation of the values reveals the variability, and at once the question arises, "What is the correct value?" A little skillful direction by the teacher can result in general agreement on the following principles:

(1) In any measurement the exact value is never known.
(2) Errors are different from mistakes.
(3) Both instrumental errors and personal errors are involved.
(4) The results of several trials indicate a central tendency. This is the "best" value we can obtain.

Without concern for the statistical treatment required, an estimate of the "likely error" may be agreed upon. Thus the distance h might be expressed as (43 ± 1) feet, and the distance x as (74 ± 2) feet. The important result in the mind of the student is that these distances are truly approximate numbers.

The data can be used for an inductive approach to several propositions. For example, the values of a and b and the ratios between certain sides of the triangles can be computed. Similar triangles can thus be discovered, and the resulting theorems, such as the relationship between the altitude and the segments of the hypotenuse, can be formulated.

Some teachers might object to the inclusion of the topic of computation with approximate numbers in plane geometry. It would seem defensible that if field work and indirect measurement, applying some of the basic principles of geometry, have a place in the curriculum, then approximate numbers are inescapable and both logically and psychologically the topic merits consideration. Certainly the student who measures the height of a flagpole and reports a value of 56.378 feet has missed an important part of the problem.

General mathematics in the secondary school—I

WILLIAM DAVID REEVE, *Emeritus Professor of Mathematics, Teachers College, Columbia University,* *recounts the development of general mathematics in the secondary school and offers suggestions for strengthening the movement toward general mathematics. Dr. Reeve's suggestions for the organization of the mathematics course at the various grade levels in the secondary school will be of interest to the general student of mathematics education.*

I. INTRODUCTION

NOTHING IN MATHEMATICS except arithmetic is offered in many secondary schools until the ninth year, where algebra is the normal course. In the tenth grade geometry constitutes the offering, and from there on there is no established pattern.

In some schools the second course in algebra (intermediate algebra) is offered in the tenth grade, and geometry is postponed until the eleventh year. Sometimes intermediate algebra, trigonometry, solid geometry, college algebra, or some other subject is offered in the third or fourth years, but in many schools practically no mathematics is offered in either of these years. If anything is offered in these later years, the content material is determined largely by individual teachers according to their own particular interests.

If one picks up almost any current intermediate algebra text, he will find that most of the book is a review of elementary algebra. It is all right to go back to the old material (elementary algebra) in order properly to connect up with the new material (intermediate algebra), but needless repetition should certainly be avoided. One can find a similar situation in the current textbooks in college algebra, where he will read page after page before he finds anything that can properly qualify as college algebra. Such extended repetition is not only bad, but unnecessary. It practically says to the student, "You don't have to learn it the first or second time over, we will give it all to you in the next course." It is needless to say that such practice has had a bad psychological effect on the students.

We now know, as a result of our knowledge of individual differences of ability, that the present practice of teaching such varying types of students as are found in the same class in some schools will, if persisted in, finally lead to educational bankruptcy. It is inevitable, therefore, that certain remedial adjustments must be made. Within reasonable limits we must abandon the ironclad traditions governing group instruction.

The Commission on Post-War Plans insists that

> ***The sequential courses should be greatly improved.*** The mathematical requirements of technology are far greater than has been generally assumed. Leadership in these fields presupposes

an extent and range of mathematical scholarship far greater than schools have traditionally made available. A fraction of the schools' student body—unfortunately no one knows how large—will need more mathematics than educators, or even mathematics teachers, have realized.[1]

In its Second Report the Commission stated 34 theses, of which thesis 14 to thesis 19 inclusive dealt directly with sequential courses for grades 10 to 12 as follows:

Thesis 14. *The sequential courses should be reserved for those pupils who, having the requisite ability, desire or have need for such work.*

Thesis 15. *Teachers of traditional sequential courses must emphasize functional competence in mathematics.*

Thesis 16. *The main objective of the sequential courses should be to develop mathematical power.*

Thesis 17. *The work of each year should be organized into a few large units built around key concepts and fundamental principles.*

Thesis 18. *Simple and sensible applications to many fields must appear much more frequently in the sequential courses than they have in the past.*

Thesis 19. *New and better courses should be provided in the high schools for a large fraction of the school's population whose mathematical needs are not well met in the traditional sequential courses."*[2]

The Commission further said:

When taught by good teachers to qualified students, the sequential courses are as well liked as any that are offered by our schools. Even before the war got under way a *Fortune*[3] poll showed mathematics to be the best liked subject in the high school curriculum. (Note well—also the least liked.) Military authorities are on record to the effect that the product of the four years' sequence is highly satisfactory as a pool of leadership for the armed forces.[4]

The Commission also maintained:

No one should assume that all is well with the traditional courses. In most schools, they are woefully out of date, as regards both subject matter and method.[5]

The Commission uses the term "traditional courses" to mean the same as "sequential courses" which is all right if that is what they mean. Other writers use the term "college-preparatory courses." Evidently, the Commission is justified in appealing as it did for better names for courses which will meet the varying needs of the students whom they are to serve. The thing that disturbs me most is that none of the names that have been used to describe various types of courses mean the same thing to all teachers and educators.

I have no desire to discredit the traditional organization of mathematics where the results have been good. I know, as a matter of fact, that, for many students, the sequence of algebra, geometry, trigonometry, and so on will continue to be that chosen in an appreciable number of schools, for reasons that may seem valid enough, at least to a great many teachers. If we can agree on that, we may then view honestly and fairly the question as to whether, for large numbers of students, an extended course in general mathematics may not be more profitable.

Teachers generally have agreed with the facts referred to above, but they have been rather slow in changing their courses to agree with them. We shall continue to make progress along this line, but we must not expect a large part of the teaching body to assimilate new material very fast. On the other hand, those who have been connected with experimental schools of one kind or another have come to feel more deeply than ever before the responsibility that rests upon their shoulders to continue to do pioneer work.

II. THE HISTORY OF THE GENERAL MATHEMATICS MOVEMENT

The meaning of general mathematics

By general mathematics, I mean a completely reorganized course in informal geometry, arithmetic, algebra, demonstrative geometry, trigonometry, analytical geometry, and the calculus beginning in the seventh grade and running continuously through the secondary school, and

[1] "The First Report of the Commission on Post-War Plans," The Mathematics Teacher (May 1944), pp. 230–31.

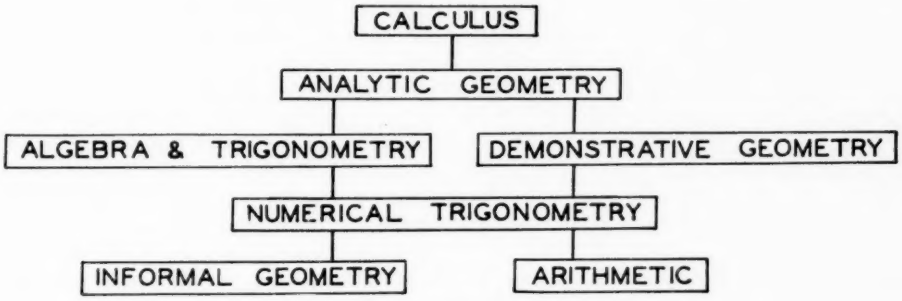
[2] "The Second Report of the Commission on Post-War Plans," The Mathematics Teacher (May 1945), pp. 208–11.

[3] "How Popular Is Mathematics Among High School Pupils?" *Fortune Magazine*, November and December, 1942.

[4] *First Report*, Commission on Post-War Plans, *op. cit.*, p. 230.

[5] *Ibid.*, p. 232.

organized in such a way as to show how these various subjects are related, in order that they may reinforce and supplement each other in useful ways. The following chart shows how the various parts are related:



The General Mathematics Tree

Such reorganization and correlation of the various subjects as I have indicated above has already occurred to some extent in the seventh and eighth grades. If carried on into grades 9–12 it will make mathematics not only more interesting and meaningful, but it will give a broader and more comprehensive view of the subject as a whole. Moreover, it will also help greatly to avoid the excessive waste that results when material is repeated from one year to another as it is done in the traditional courses where there is little if any correlation. Some repetition, of course, is both necessary and desirable if it is not carried too far. If the spiral plan of teaching is used, and it has been found to be a good plan, we shall need to look back occasionally and consider what has already been covered.

Such a scheme will give the student who desires it a six-year course in mathematics in the secondary school. At the end of such a course, he will be as well prepared as most college freshmen are today, if not better. The course will also develop more power, and produce even better results than we have been getting from traditional methods.

Facts connected with the general mathematics movement

Let us now consider some interesting facts connected with the general mathematics movement. The idea of general mathematics is not new. The artificial pigeon-holing of subject matter is a practice which many good teachers have never observed. In other words, there is nothing essentially new in the idea of trying to correlate one subject with another. The best teaching talent in mathematics the world over long ago recognized the importance of emphasizing the relation between algebra and geometry and the other subjects, and the advisability of teaching them together. The movement to unify the various subjects has been vigorously opposed by those who seriously insist that such unification will break up the logically ordered system of Euclidean geometry and give rise to a series of unrelated ideas without unity or natural sequence. However, this is not true, since we do not try to correlate algebra with the demonstrative aspects of formal geometry. Besides, if we are ever to cut the "ancestral process" short enough to enable us to give the secondary students anything beyond what we ourselves had, we must omit a great deal of the now obsolete material and reorganize what is left in a more psychological and pedagogical manner.

John Perry's influence

John Perry, many years ago in England, saw the importance of reorganizing content and method. He put the case as follows:

> Great fields of thought are now open which were unknown to the Alexandrian philosophers. If we begin our study as the Alexandrian philosophers did, with their simplest ideas in arithmetic and geometry, we shall get stale before we know much more than they did. If we begin assuming more complex things to be true (although I do not like to assume that in truth any idea is more complex than another) as we have done in arithmetic, as we ought to do in other parts of mathematics without becoming stale, we may know of all the modern discoveries. We shall get the same intellectual training with more knowledge.[6]

The importance of less formality

Classroom experience shows that when certain things in mathematics are necessary to the student's progress they should be given him, if need be, without proof. He should be taught how to use them until they become a part of his machinery, and later, when he is more mature, he can study further and develop the nature of the machinery he has been using. This is what we all do every day. We simply take on faith a number of things we cannot fully understand. We can thus lead the student into much higher and more powerful mathematics without any ultimate loss. It is well known, in the study of mathematics, that we continually make large assumptions where failure to do so would be not only unwise but ridiculous.

The important thing to consider in the development of a general mathematics course is psychological order rather than logical order; a longer time exposure which will minimize the student's difficulties and which will avoid the serious mistake of forcing deductive logic at too early an age. In such an organization of mathematics it is conviction that we want at first; not proof in the formal sense. For example, the student is not expected to prove that "if two lines intersect, the vertical angles thus formed are equal." He knows by his intuition that the statement is true, and no amount of formal proof will be any more convincing.[7]

The very nature of a general mathematics course, as I define it, makes it more interesting and, in actual practice, more easily learned than the traditional algebra or geometry course. General mathematics, if properly organized, would be more interesting and valuable to both the gifted students and the slow learners.

Outside of Perry's influence in this country, there was the impetus given the reorganization movement by the presidential address delivered by E. H. Moore,[8] of the University of Chicago, before the American Mathematical Society at its annual meeting in 1902. Through Moore's influence, and that of G. W. Myers of the School of Education at Chicago, E. R. Breslich and his associates in the mathematics department of the University of Chicago High School carried on experiments and published some of the earliest texts on correlated mathematics.[9]

I first became interested in general mathematics in 1908 when I was a student in a course given by J. W. A. Young at the University of Chicago. I became a member of the mathematics department at the University of Chicago High School in 1910 and assisted in the experiment that was carried on there in an attempt to correlate mathematics in grades 9–12.

In 1915 I went to the University of Minnesota High School as head of the mathematics department where, from

[6] *Discussion on the Teaching of Mathematics*, ed. John Perry (New York: Macmillan and Co., 1902), p. 13.

[7] G. St. L. Carson, *Essays on Mathematical Education* (Boston: Ginn and Company, 1913), pp. 17–32. An excellent essay on "Intuition."

[8] E. H. Moore, "On the Foundation of Mathematics," *A General Survey of Progress in the Last Twenty-Five Years* (First Yearbook, National Council of Teachers of Mathematics, 1926), pp. 32–57.

[9] There were two books by Myers and his associates called:
(1) *First Year Mathematics* (1906)
(2) *Second Year Mathematics* (1910)
Breslich later revised these two books and wrote a third book to complete the series. These books were all published by the University of Chicago Press.

1915 to 1923, I worked out a four year course in general mathematics. Two books were published during that period.[10] I came to Teachers College in 1923, and there, in the Horace Mann School for Girls, I continued the experiment in general mathematics for six years more. I took a class of students at the beginning of the seventh grade and took them through six years of work until they graduated. Incidentally, all of the students who were in this group and had to take college entrance examinations in mathematics, passed those examinations without any difficulty. Moreover, these students were not continually reminded all through their course that they had to pass college entrance examinations, and they certainly were not trained specifically to pass these examinations, as is true in so many schools.

My experience certainly has not proved that general mathematics is superior to the college entrance type of arrangement, but I think it does show that students can study general mathematics and still pass the college entrance examinations in mathematics without becoming emotionally upset as is often the case.

The work of The National Committee on Mathematical Requirements

The National Committee on Mathematical Requirements, in discussing "general" courses, said:

> In recent years there has developed among progressive teachers a very significant movement away from the older rigid division into "subjects" such as arithmetic, algebra, and geometry, each of which shall be "completed" before another is begun, and toward a rational breaking down of the barriers separating these subjects, in the interest of an organization of subject-matter that will offer a psychologically and pedagogically more effective approach to the study of mathematics.
>
> There has thus developed the movement toward what are variously called "composite," "correlated," "unified," or "fused," "general" courses. The advocates of this new method of organization base their claims on the obvious and important interrelations between arithmetic, algebra, and geometry (mainly intuitive), which the student must grasp before he can gain any real insight into mathematical methods and which are inevitably obscured by a strict adherence to the conception of separate "subjects." The movement has gained considerable new impetus by the growth of the junior high school, and there can be little question that the results already achieved by those who are experimenting with the new methods of organization warrant the abandonment of the extreme "watertight compartment" method of presentation.
>
> The newer method of organization enables the pupil to gain a broad view of the whole field of elementary mathematics early in his high-school course. In view of the very large number of pupils who drop out of school at the end of the eighth or the ninth school year or who for other reasons then cease their study of mathematics, this fact offers a weighty advantage over the older type of organization under which the pupils studied algebra alone during the ninth school year, to the complete exclusion of all contact with geometry.[11]

The argument given by The National Committee on Mathematical Requirements for a general course in mathematics in secondary education is as potent today as it was in 1923.

McCormick's study[12]

We know from careful research done by McCormick twenty-five years ago that a one-year course in general mathematics is no better than a course in algebra. However, results have shown that a two-year course in general mathematics is the equal if not the superior to a year of algebra followed by a course in geometry for the students concerned. Those of us who have taught such courses know that this is true. A three year course would be still better for those who are able to pursue it.

[10] Raleigh Schorling and W. D. Reeve, *General Mathematics, Book I* (1919), and W. D. Reeve, *General Mathematics, Book II* (1923). Both of these books were published by Ginn and Company, Boston.

[11] *The Reorganization of Mathematics in Secondary Education* (a Report by The National Committee on Mathematical Requirements under the auspices of the Mathematical Association of America, Inc., 1923), pp. 12–13. A later edition of this report was published by Houghton Mifflin Company, Boston, 1927.

[12] Clarence McCormick, *The Teaching of General Mathematics in the Secondary Schools of the United States* (New York: Contributions to Education, No. 386, Bureau of Publications, Teachers College, Columbia University, 1929).

When McCormick made his study, he was comparing the traditional courses with general mathematics as I have defined it. Little research since McCormick's investigation has been carried on with the consumer type of general mathematics, as a later investigator has done,[13] to try to determine the superiority of general mathematics, if any, over the traditional courses. One reason for this is the fact that the leaders in the general mathematics movement did not prepare suitable textbooks along that line. As a result, textbooks on so-called consumer mathematics and the like began to appear, because teachers felt that the algebra of the traditional course was too hard for many of their students. The result has been that we have drifted along, and a great deal of confusion has resulted as to what general mathematics really is. This can also be seen by examining recent state and regional courses of study.

However, this cannot be said of the work of the seventh and eighth grades, where general mathematics constitutes the core of the curriculum.

The work of John Swenson

John Swenson, a teacher in Wadleigh High School, New York City, who told me he was greatly influenced in his thinking by the work of The National Committee on Mathematical Requirements, did a great deal of experimental work in teaching general mathematics at Wadleigh High School and later elsewhere in the city. As head of the department and therefore a member of the New York City chairmen's organization, Swenson exerted great influence in the metropolitan community, and a great deal of the subsequent experimental work done by other teachers is largely due to his efforts. He gave a course in professionalized subject matter for many years at Teachers College, Columbia University, and thus was able to influence the thinking of a large number of teachers who registered for his course. Swenson subsequently published a series of four books for grades 9–12 which he called "Integrated Mathematics."[14] They were general mathematics, just the same as I have defined it.

The work of the Joint Commission

The Joint Commission of the Mathematical Association of America and the National Council of Teachers of Mathematics issued a report in 1940[15] which outlined two alternative plans for introducing general mathematics into the secondary curriculum.

III. THE PRINCIPLES OF ORGANIZATION OF THE GENERAL MATHEMATICS COURSE

Traditional courses in mathematics have delayed the presentation and consideration of much interesting and valuable material which may well be used to give the student, early in his career, an idea of what mathematics means and something of the wonderful scope of its applications. Trigonometry, which contains many real problems of interest to the student and easily understood by him, furnishes an interesting example of this delay. Other examples are found in the use of logarithms as a labor-saving device, the slide rule, standardized graphical methods, computation with approximate numbers, the common constructions of practical drawing, the function idea, and the introductory ideas of analytic geometry and the calculus.

The student should be given an opportunity to use these tools early in his study of mathematics. They lend to the subject a power and interest that drills on formal material cannot possibly give. We can

[13] M. W. Beckmann, *The Level of Mathematical Competency and Relative Gains in Competency of Pupils Enrolled in Algebra and General Mathematics* (Unpublished Ph.D. dissertation, Dept. of Secondary Education, University of Nebraska, 1951).

[14] J. A. Swenson, *Integrated Mathematics*, Books I–IV (Ann Arbor, Michigan: Edwards Brothers, 1937).

[15] *The Place of Mathematics in Modern Education* (The Fifteenth Yearbook, National Council of Mathematics, 1940), pp. 72–112.

gain time for the study of these new topics by effecting economies all along the line.

Many teachers have been spending a whole semester on material that ought to be covered in considerably less time. In this connection I want to suggest that great economy in time could be achieved if only we would determine, scientifically, how long it takes to teach certain topics satisfactorily. And by satisfactorily we should mean "to teach the topic to the degree of perfection indicated by some standard agreed upon." For example, how long does it take a normal group to learn how to factor the difference of two squares so that nobody in the group will make more than one error out of fifteen possible examples?

Many experiments have been made to determine the best general method of presenting mathematics. From such efforts have come certain principles which determine the procedure as follows:

1. The first of these principles is a very old one; namely, that the work should proceed from the simple to the complex. In general mathematics this principle is much broader in application than it is in the traditional courses where the complex parts of algebra, for example, are taught before the simple parts of geometry.

2. The second principle is also an old one, as are substantially all the principles of successful teaching. It maintains that the student should proceed gradually from the familiar to the new by means of easy steps which bring out the relation of what he already knows to the new field upon which he is entering.

3. The third principle is that a student is more successful if he studies one thing at a time, and for a sufficient length of time to acquire at least some feeling of mastery.

As any successful teacher knows, a proper view of the nature of correlation seeks points of genuine contact between any two parts of the science. The fact that two branches of study are both classified under the general name of mathematics is no reason for assuming that there is any close relation between them. Authors of textbooks should not seek, merely for the sake of novelty, to mix the various topics of a science in which compartments naturally exist. They should have perfect sympathy with scientific correlation, and should adopt the idea, but they should have no sympathy whatever with an unnatural and unscientific mixing or fusing of unrelated topics.

4. The fourth principle is that a student has the right to see the genuine practical applications of the mathematics which he studies, and that he has an equal right not to have thrust upon him a mass of applications which he will realize are merely made up for the occasion in order to lend an appearance of reality to the work. Authors and teachers should therefore be perfectly frank with the student by stating that certain parts of algebra, for example, are not utilitarian in themselves, but are necessary to enable him to understand the methods of solving the technical problems which represent the practical uses of the topic, but which are too advanced to be considered at the time. It is much more important that the student gain some appreciation of the significance of the fundamental elements of solid geometry and of trigonometric methods than that he should attempt to cover all the plane geometry that could be given in a one-year course. The work in these topics is not so difficult as the parts of algebra and geometry which it displaces, and it is much more interesting to the student.

The fact is that some algebra and geometry textbooks today are presenting more and more general mathematics in line with the above definition. But progress is slow because of the ever-present time lag in American education. The case for general mathematics rests largely on the fact that it gives results that are highly gratifying. This is especially true if the work is continued throughout the entire secondary school, or even through the eleventh year.

IV. THE CURRICULUM IN GENERAL MATHEMATICS

The construction of a curriculum in general mathematics involves the consideration of the wide divergencies that exist, at the present time, both in the elementary school and in the purposes of secondary school instruction itself. Thus, for some students the work of the elementary school may have included the usual course in arithmetic followed by a year of general mathematics or some other course, or it may have included courses of the type offered in the three years of the modern junior high school, or a combination of these courses. Furthermore, in the senior high school some students may be preparing to study in technical schools or colleges, while others may propose to establish themselves at once in a business or other career. Everybody should know something of the essential applications of arithmetic, of the central features of informal geometry, of the meaning and uses of algebra, of the nature and value of numerical trigonometry, and of the meaning of a demonstration in geometry. Such a course should be required of all students, whether they leave school at the end of the ninth grade or go on to the senior high school.

While no single order of topics or particular grade placement[16] of materials is absolutely essential, there has come to be a fairly uniform practice in the placing of the larger units within the various years. The following outline suggests what has proved in actual practice to be successful.

Grade 7	*Grade 8*
Informal geometry	Algebra
Arithmetic	Business arithmetic
Algebra	Informal geometry

Grade 9

Algebra (beyond mere utility)
Numerical trigonometry
Demonstrative geometry (brief)
Arithmetic

No matter how the material is arranged, if we follow this general scheme, we may expect at least two important outcomes. First, the student will have the best possible preparation, within the time allowed, to meet the mathematical needs that confront him in practical life. Second, he will be preparing himself in the best possible way to continue his work in mathematics and science if he proceeds further in school.

What the students know at the beginning of the seventh grade

Arithmetic is finished at the end of the sixth grade in practically all countries except the United States, in so far as the fundamental skill operations or essential parts of computation with whole numbers and fractions are concerned. There are, of course, very many practical applications which are too hard for students to understand in the sixth grade. We should include some of them in the junior high school, but we should not include them all.

(*To be concluded in the March issue*)

[16] *The Place of Mathematics in Modern Education* (Fifteenth Yearbook, National Council of Teachers of Mathematics, 1940), pp. 246–251.

Anything funny ever happen in your class?

Z. L. Loflin, Southwestern Louisiana Institute, Lafayette, Louisiana, a member of the Editorial Board of THE MATHEMATICS TEACHER, is interested in receiving jokes, cartoons, etc. that have a mathematical point. "Zeke" has accepted the responsibility of gathering those funny incidents about mathematics that all classroom teachers encounter from time to time. Why not share your laughs with the readers of THE MATHEMATICS TEACHER by sending them to the above address.

The pupil discovers algebra

EUNICE LEWIS, *Supervisor of Mathematics, University High School, Norman, Oklahoma,* *argues that the inductive method of instruction should predominate in algebra classes. Several illustrations are provided to show how algebra can be taught inductively.*

YOU ARE WALKING down the street when a black cat crosses your path. You would like to alter that path if it weren't for the fact that you are an educated person and are not supposed to believe that black cats bring disaster. While you are thinking how you might unobtrusively alter your course, remember that there are ever so many other people in your situation who would like to avoid black cats in order to avert impending catastrophe. This, we recognize, is a superstition. It developed, no doubt, from coincidental misfortunes attending those whose paths had been crossed by a black cat. A generalization was made from these few experiences, hence the superstition was born.

Generalization, a result of induction, has its origin in the first speculations of mankind. However, the many examples of generalizations made today often lead one to suspect the validity of such a process. That is, women buy the soaps and smoke the cigarettes used by glamorous movie stars; men buy the hair lotions which have retained hair on otherwise bald pates; and everyone clamors for that health tonic which is a sure cure of all ailments and the elixir of youth. These generalizations, along with our prejudices and superstitions, have grown from superficial acquaintances and the observation of too few instances or cases. When certain qualities have been observed in a few cases, and all other such cases are expected to conform to these same qualities, then the uncritical mind is at work. In developing the growth of our scientific attitude, we must hold a tight checkrein on our impulsive inclination to arrive at a conclusion from too few observations. We must progress cautiously, testing, evaluating, rejecting; and we must constantly be aware that our discoveries are characterized by a high degree of probability.

From the foregoing discussion, it seems as if the inductive method has been fully convicted, and a suitable sentence should be meted out to all such trouble makers. Would this be good? Suppose in the beginning one edict had been handed down to man—that at no time was he ever to use the inductive method in his search for knowledge, in his attempt to understand the world in which he lived. Obviously, we would be living in a world with no understanding of the great forces acting in our universe, such as the laws of growth, of gravitation, of energy; no scientific discoveries in the fields of medicine and science. There would have been a definite postponement in the development of the laws of algebra. On the whole, the progress of our civilization has come through an inductive process.

Since the inductive method plays such an important role in our attempt to interpret the world about us, why should not our pupils learn more about this valuable method of thinking early in their mathematical training? Also, due to the elements of discovery and exploration which characterize the inductive approach, no adolescent should be deprived of this adventure. According to Miss Helen M. Walker, who writes in the

Seventh Yearbook of the National Council of Teachers of Mathematics, it is our responsibility as teachers to make it possible for each pupil "to feel the thrill of mathematical pioneering." She believes that "the casualties will be fewer and less serious" when the "spirit of discovery" is allowed to predominate in learning situations.[1] And, according to the *Second Report of the Post War Commission*, "a pupil learns quickly and remembers longer the things that he understands fully,"[2] hence more emphasis must be given to the development of meanings. It is quite evident that, when a pupil discovers relationships, he is likely to understand them, and in turn they become more meaningful to him.

Briefly, the inductive approach is that technique of making a transition from the particular to the general—from facts which have been discovered through observation and experiment to generalizations. There are certain basic steps which we must necessarily observe when arriving at a generalization. They are: collection of data through observation and experiment, organization of the data, formation of an hypothesis, verification of the hypothesis by further observation and experiment, and finally the formulation of a generalized statement which will adequately express the relationships observed from the collected data.

Now, up to the time the pupil is introduced to algebra, his mathematical experiences have been with particular, concrete situations, with little or no generalization involved. Hence, the natural approach to algebra, an entirely new concept, should be made through the medium which he knows best, the particular situation. Walter W. Hart says that "the inductive method of instruction should predominate in elementary algebra."[3] Students look on algebra more favorably when told that it "is a series of discoveries in which they can share."[4]

Recognizing that the use of this procedure cannot and should not be used in certain areas of algebra, let us consider a few situations in which the inductive method may effectively be used. Helping the pupils discover the laws governing the multiplication and the division of terms containing exponents is probably more widely practiced than any other topic. Using the definition of an exponent, the pupil is asked to express $(x^3)(x^2)$ in factored form, as

$$(x^3)(x^2) = (x \cdot x \cdot x)(x \cdot x)$$
$$= x^5, \text{ by reversing the definition.}$$

After the pupils have been given several examples of this type, using different exponents each time, some pupil will inform his classmates that he can get the answer simply by adding the exponents. Here the teacher picks up her cue, capitalizes on this "discovery," and assists the "explorer" to express his new idea in the form,

$$(x^3)(x^2) = x^{3+2}.$$

After the pupils have expressed the other examples in this form, they will be ready to offer the following generalization,

$$(x^m)(x^n) = x^{m+n}.$$

Similarly, the law governing the division of numbers written in exponential form can be "discovered" by any class that is encouraged in, and given an opportunity for, mathematical exploring.

Suppose we next consider the graphing of the linear equation in the slope-intercept form. Assign to be graphed several families of equations, each family being placed on the same set of axes. Let the

[1] Helen M. Walker, "Adventures in Algebra," *Seventh Yearbook of the National Council of Teachers of Mathematics* (New York: Bureau of Publications, Teachers College, Columbia University), p. 138.

[2] "The Second Report of the Commission on Post-War Plans," THE MATHEMATICS TEACHER, XXXVIII (1945), p. 208.

[3] Walter W. Hart, *Teacher's Manual for a First Course in Algebra* (D. C. Heath Co.), p. 7.

[4] Carl Denbow, "Teaching the Laws of Algebra, or Mathematics Is a Language," THE MATHEMATICS TEACHER, XXXIV (1941), p. 361.

families consist of equations having different slopes and passing through the origin, and of equations having the same slope with different points of intersection on the Y-axis. Families of the first type could be,

$$\begin{array}{lll} y=x & y=-x & y=x \\ y=2x & y=-2x & y=3x \\ y=\frac{1}{2}x & y=-\frac{1}{2}x & y=\frac{2}{3}x \end{array}$$

Families of the second type could be,

$$\begin{array}{lll} y=2x & y=x & y=\frac{1}{2}x \\ y=2x+1 & y=x+1 & y=\frac{1}{2}x+2 \\ y=2x-1 & y=x-2 & y=\frac{1}{2}x-3 \end{array}$$

It is most necessary that a sufficient number of examples be used before any generalizations can be expected. If the spirit of adventuring has been made a predominant attitude in the class, teacher guidance will be at a minimum.

Now the fun begins! Similarities are noted between the graphs and the equations which they represent, and also between the graphs of a family and their equations. What high school pupil doesn't have fun when he can "tell" the teacher? He will tell you that the coefficient of the X has "something to do with" the slope, and that the graph crosses the Y-axis at a point which is the same as the constant term in the equation. Now it is up to you to direct his thinking. Assist him in analyzing, then in generalizing. The pupils will experience a thrill which is always associated with the realization that they have discovered something new. And the satisfaction of having made a discovery is not altered in the least by the fact that others before have made the same discoveries.

The introduction to equations might be begun with this puzzle, "I am thinking of a number which when added to 11 makes 21. What is the number?" A few pupils will tell you immediately the answer to your puzzle. After giving them several such puzzles, there are still a few pupils who have not yet "caught on." This is an ideal time for the introduction of a pair of balance scales. In one pan you place an object whose weight is 10 grams, but is unknown to the class. To this is added weights amounting to 11 grams. In the other pan you place weights amounting to 21 grams. Using groups of pupils at a time, you ask them to devise a method to find the weight of the object. This will soon be discovered, and the answer to the puzzle determined. In arriving at this answer, the pupils have observed that the scales must be kept in balance at all times, as well as the manner in which this is accomplished. It is then explained that an equation, similar to a pair of scales, has the property of balance, which must be maintained at all times. The class is guided next into expressing the puzzle in symbols, hence, $n+11=21$.

Now the class is ready for more complicated puzzles, such as, "If I should double my present weight, then gain 10 pounds, the result would be 270 pounds. Who can tell me how much I weigh now?" After some analysis, the class will discover that the process of determining my weight is one of "undoing" or "unraveling." The pupils will have little difficulty in translating the puzzle into the algebraic symbols, $2x+10=270$. They will need assistance at first in the process of "undoing." Soon they will be explaining in a most discerning manner that since 10 has been added to $2x$, we "undo" this by subtracting it, but to maintain the balance, the 10 must be subtracted from both sides, as,

$$\begin{aligned} 2x+10 &= 270 \\ 2x+10-10 &= 270-10 \\ 2x &= 260 \end{aligned}$$

Continuing with the "undoing" process, they will recognize that x has been multiplied by 2. This, they will say, is "undone" by dividing by 2, but maintaining the balance requires that both sides of the equation be divided by 2. Hence,

$$\begin{aligned} 2x &= 260 \\ \frac{2x}{2} &= \frac{260}{2} \\ x &= 130 \text{ lb.} \end{aligned}$$

The pupils have discovered teacher's weight, but how much more fun it would have been to have discovered her age! Nevertheless, the spirit of being allowed to seek and find was fun. At this stage there is no need to burden the pupils with the formal statements of the axioms of multiplication, division, addition, and subtraction. This will come later. The main concern of the class at the present is to find the answer to the puzzle, which they have discovered can be accomplished by the "undoing" process. However, during the performance of this process, they have become aware of this informal generalization, that if a number is to be added to, subtracted from, multiplied by, or divided into one side of an equation, the same must be done to the other.

Why should pupils be told the laws governing the addition and subtraction of directed numbers, when they can discover them? Since most modern algebra texts suggest that these two laws be developed inductively through the use of the directed number scale, time will not be taken to discuss this topic. However, suggested variations are: to convert the scale into a football field with the team gaining and losing yardage, a highway with cars traveling to and from towns and passing each other, an elevator which makes stops at levels below the first floor as well as above, and many more which could be cited. The success of this approach depends upon giving the pupils sufficient experience from an adequate number of examples or cases before calling on them for a generalization. This is where most teachers fail. Too frequently they hurry on, giving the generalization themselves, leaving the pupils in a state of utter confusion. Never rush generalizations. Give the pupils time to generalize, and let them realize that you have confidence in their ability to do so.

An approach not usually found in textbooks is one concerning the division of a polynomial by a binomial. Being familiar with their arithmetic, the pupils will have a feeling of security when asked to divide 156 by 12. After performing this division, the pupils will be ready to further extend this ability. Now discuss the way a number is formed when its digits are known, and suggest that 156 and 12 be expressed using this idea. The problem will appear in this form,

$$(1(10)+2)\,\overline{|1(100)+5(10)+6.}$$

Expressing 100 as $(10)^2$, the solution follows,

$$\begin{array}{r} 1(10)\ +3 \\ \hline 1(10)+2\ \big|\ 1(10)^2+5(10)+6 \\ \underline{1(10)^2+2(10)\ \ \ } \\ 3(10)+6 \\ \underline{3(10)+6} \end{array}$$

Next suggest that the (10) be replaced with the letter (x). Hence the generalization,

$$\begin{array}{r} 1(x)\ +3 \\ \hline 1(x)+2\ \big|\ 1(x)^2+5(x)+6 \\ \underline{1(x)^2+2(x)\ \ \ } \\ 3(x)+6 \\ \underline{3(x)+6} \end{array}$$

This gradual transition from the particular to the general is natural, and the pupils make the adjustment readily and with ease.

Similarly, the product of 13 and 12 is expressed as,

$$\begin{array}{r} 1(10)+3 \\ \underline{1(10)+2} \end{array}$$

which generalizes into the product of $(x+3)$ and $(x+2)$.

The quadratic equation is a topic which offers many possibilities in the realm of mathematical adventuring. Assign to the class several problems of the type $ax^2+bx+c=0$, asking that the roots of each be found by both the quadratic formula and by graphing. With a little guidance, some one will soon observe that the expression

$-b/2a$ in the formula is the x coordinate of the maximum or minimum point of the curve of $y=ax^2+bx+c$, and that $x=-b/2a$ is the axis of symmetry of the curve.

Now the stage is set and exploring will continue. A pupil's question, "How do we find imaginary roots by the graphing method?" may start a few in the class off on such an investigation. Excitement runs high when some one reports that he has found a way to do that very thing. From his observation of numerous solutions, he has discovered a relationship between the coordinates of the maximum or minimum point of the curve and the roots of the equation. The important element in this development is not the "discovery" but the "discoverer." Here he has been introduced to the scientific method of thinking —that type of thinking which is so necessary to the growth of our civilization.

Never tell a class that the nature of the roots is dependent upon the value of the discriminant of the equation. Give them a sufficient number of quadratic equations to solve, using the formula, which will yield roots that are equal or unequal, rational or irrational, and real or imaginary. A wide-awake class, full of the spirit of adventure and prodded along by a teacher endowed with the same adventuresome attitude, will make its own discovery.

Frequently a pupil makes such statements as,

$$\sqrt{a^2+b^2}=a+b \quad \text{or} \quad x^2+y^2=(x+y)^2.$$

He should be taught to test the validity of such statements by substituting various numbers for these literal values. He will be much more convinced if he discovers his own errors than if you attempt to tell him, and the learning will be more permanent in nature, also.

This inductive method is most effective in introducing functional relationships and the construction of formulas from tabular data. However, any teacher will find little difficulty in selecting other topics suited to this approach.

Indeed, the role of the teacher must not be minimized. She is the one first to establish the atmosphere of the class, the spirit of pioneering and discovery. In her hands rests the success or failure of such an approach. It is she who must realize when it is important "to withhold relevant information; when to feign ignorance or forgetfulness of some important mathematical fact."[5] It has been said that "if we were not in too great a hurry to get to mechanical devices in algebra—any child could learn the principles of correct generalizations."[6] This is so often the error committed by the teacher. Too frequently she rushes the pupils into generalizations for which they are not ready. She must be willing to accept slow progress and crude generalizations upon the first attempt. It is then her responsibility, through wise questioning tempered with patience, to guide the pupil to refine these statements into well defined generalizations. After all, pupils enjoy thinking, and appreciate being given that opportunity. A teacher must recognize this quality, and must have faith in their ability to perform this function with credit.

It is agreed that the "telling them" method can be used with less effort and with less consumption of time. It is also much easier to execute, and requires far less preparation. However, our chief concern is the pupil who is doing the learning. Which method will do the most for him? Shall he be allowed to pioneer, discover, and create through wise guidance, thus learning to appreciate the role of the scientific method in our democratic society; or shall we play the role of the dictator and "tell him" those many things which he could so well discover for himself? After all, what modern adolescent wants to be told when he can find out for himself? And —who dares "tell" the American youth of today?

[5] Helen M. Walker, "Adventures in Algebra," *The Seventh Yearbook, National Council of Teachers of Mathematics* (New York: Bureau of Publications, Teachers College, Columbia University), p. 141.

[6] Susanne K. Langer, "Algebra and Reason," THE MATHEMATICS TEACHER, XXIV (1931), p. 291.

Some classroom problems from the field of atomic energy

WALLACE MANHEIMER, *Franklin K. Lane High School, Brooklyn, New York.* *A teacher of mathematics offers suggestions on a topic of much general interest to all Americans. He develops those aspects of the subject which can be used in the mathematics classroom.*

THE RAPIDLY GROWING field of atomic energy offers striking opportunities to mathematics teachers for motivation and application of their subject. Our pupils are vitally interested in this field, not only because of its awesome possibilities for good or evil, but also because of its great vocational opportunities. Atomic energy has already become one of the greatest industries in the nation; and one that demands of its workers a high degree of mathematical understanding and technical training.

The problems described in this article were studied in classes in intermediate algebra at our school. In the opinion of both teachers and students they were not only interesting on their own account, but effective in imparting deeper comprehension and richer appreciation of mathematical concepts. Brief introductions will be given to most of the problems; however, teachers are urged to supplement this by referring to some of the excellent material that has been written on atomic energy. Obviously the problems that follow are but a small sample of one way in which we can tap the interest of our pupils in an important phase of modern life.

I. REPRESENTATION OF NUMBERS IN EXPONENTIAL FORM

The number, 1.35×10^{-8}, means more to our pupils if they are told that it is the diameter, in centimeters, of the hydrogen atom. Atomic physics, of course, abounds in such numbers, both great and small. For example, the half-life intervals of radioactive elements range from 3×10^{-7} seconds for polonium 212, to 4.51×10^{9} years for uranium 238.

The problems that follow illustrate operations on numbers expressed in this form.

1. In the formula for the energy equivalent of mass, $E=mc^2$, c represents the speed of light. If m is expressed in grams and E in ergs, $c=3\times10^{10}$ cm. per sec. If, in the explosion of a Hiroshima type atom bomb, about .93 grams of matter are converted to energy, what is that energy expressed in ergs?

Solution:

$$\begin{aligned} E&=mc^2\\ &=.93\ (3\times10^{10})^2\\ &=.93\times9\times10^{20}\\ &=8.4\times10^{20}\text{ ergs.}\end{aligned}$$

2. If one calorie equals 4.2×10^{7} ergs, how many calories are released in an explosion of a Hiroshima type bomb?

Solution:

$$\begin{aligned} C&=\frac{8.4\times10^{20}}{4.2\times10^{7}}\\ &=2\times10^{13}\text{ calories.}\end{aligned}$$

3. The energy released in an explosion of a ton of TNT equals about 10^{9} calories.

Express the energy of an atom bomb in terms of tons of TNT.

Solution:

$$T=\frac{2\times10^{13}}{10^9}$$

$$=2\times10^4$$

$$=20{,}000 \text{ tons of TNT.}$$

II. GEOMETRIC PROGRESSIONS

Half life of a radioactive element

The half life of a radioactive element is defined as the period after which half of its mass will have decayed into elements lower down in the radioactive chain. Obviously the amounts that decay in successive half-life intervals form a geometric progression with ratio $\frac{1}{2}$.

1. What per cent of a radioactive element remains after a sample of it has passed through ten successive half-life intervals?

Solution:

$$R=(\tfrac{1}{2})^{10}=\frac{1}{1024}=.1\% \text{ (approximately).}$$

2. In an atomic pile, the radioactive element, neptunium 239, is produced. This substance has a half life of 2.3 days, its decay product being the fissionable element, plutonium 239, that is used in a Nagasaki type atom bomb. How long must a mass of neptunium be "processed" for 90% of it to be converted to plutonium?

Solution: The amount of neptunium that remains will be 10% of the original mass. Letting x represent the number of half-life intervals that must elapse, we obtain

$$(\tfrac{1}{2})^x=\frac{1}{10}, \quad \text{or}$$

$$2^x=10$$

$$x\log 2=\log 10$$

$$x=\frac{1}{.3010}, \quad \text{or} \quad x=3.3.$$

The answer to the problem, therefore, is 3.3×2.3 days, or 7.6 days.

3. Uranium 238 has a half life of about 4.4×10^9 years. Assuming the age of the earth to be about 2.2 billion years, and assuming a mass of pure uranium in the earth's crust at its origin, with what per cent of decay products should we expect the present day ore to be found?

Solution: 2.2 billion equals 2.2×10^9, or about one-half the half life of uranium 238. Therefore the amount of uranium remaining from a given sample can be found by multiplying by $(\frac{1}{2})^{1/2}$ or $1/\sqrt{2}$.

$$\frac{1}{\sqrt{2}}=\tfrac{1}{2}(\sqrt{2})=71\%.$$

Therefore the percentage of decay products will be about 30%.

4. Sodium 24, a radioactive isotope of sodium, has a half life of 14.8 hours. If some of this isotope remains as a dangerous radioactive after effect of an atom bomb, how many hours must elapse for 87.5% of it to be dissipated?

Solution: 12.5%, or $\frac{1}{8}$ of the original amount must remain. Since $\frac{1}{8}=(\frac{1}{2})^3$, three half-life intervals, or 44.4 hours must elapse.

5. Iodine 131, which is used in the treatment of cancer of the thyroid gland, has a half life of 8.0 days. By what per cent has the effectiveness of a quantity of this element decreased after two days?

Solution: Two days equals $\frac{1}{4}$ a half-life interval. Therefore the amount of radioactive iodine remaining will equal the original amount multiplied by $(\frac{1}{2})^{1/4}$, or .84. It follows that the original sample will have lost 16% of its effectiveness.

Multiplication factor of an atomic reactor

In an atomic reactor, or atomic pile, slow moving neutrons cause atoms of uranium 235 to undergo fission, which process releases in turn another quantity of neutrons. The ratio of the second quantity of neutrons to the first is called the "multiplication factor" of the reactor. A "self-sustaining chain reaction" is defined

as one in which the number of fissions remains the same or increases with time.

1. What will be the total (theoretical) number of neutrons set free in an atomic reactor with a multiplication factor of .7 by the application to it of 100 neutrons?

Solution: The total, including the original 100 neutrons, will be the sum to infinity of a geometric progression with first term equal to 100 and ratio equal to .7

$$s=\frac{a}{1-r}$$
$$=\frac{100}{1-.7}$$
$$=333$$

2. In the above reactor, if the multiplication factor is increased to 1.03, how many fissions will occur after a series of 35 fission interactions?

Solution:

$$s=\frac{ar^n-a}{r-1}$$
$$=\frac{100(1.03)^{35}-100}{1.03-1}$$
$$=\frac{100(2.805-1)}{.03}$$
$$=6{,}000, \text{ approximately.}$$

III. VARIATION

Critical size of a reactor or atom bomb

Although the following analysis is applied to spheres, it will be plain that it is equally true of any similar solids.

The number of neutrons available to produce fissions in a spherical reactor or atom bomb is proportional to its volume. It therefore varies as the cube of the radius. The number of neutrons that escape from it is proportional to its total area, and therefore to the square of the radius. Thus it can be seen that, as the radius increases, the ratio of the number of escaping neutrons to fission-producing neutrons will decrease. If a self-sustaining chain reaction can ultimately be set up, the point at which this occurs is called the "critical size" of the bomb or reactor.

1. If the ratio of escape neutrons to fission-producing neutrons in a sphere of uranium 235 is .08, what is this ratio in a sphere three times its size?

Solution: Calling n_e the number of escape neutrons and n_f the number of fission-producing neutrons, we have,

$$n_e=k_1r^2$$
$$n_f=k_2r^3$$
$$\frac{n_e}{n_f}=\frac{k_1}{k_2r}$$

This proves that the ratio in question varies inversely as the radius of the sphere. The answer to the problem, then, is

$$\frac{n_e}{n_f}=.027$$

2. Assume that the critical size of a reactor is reached when the ratio of escape to fission-producing neutrons is .003. If this ratio is .01 for a sphere of radius r_0, express the size of the critical radius in terms of r_0.

Solution: Using the result found in the first problem, we obtain $r/r_0=.01/.003$, or $r=3.33\ r_0$.

Separation of isotopes

If the concentrations of a substance in mixtures before and after an enrichment process are f and f' respectively, the "separation factor," s, of this process is defined by the relationship,

$$s=\frac{f'}{f}.$$

The condition for increase of concentration is, of course, that s be greater than 1. In most isotope separation processes the value of s is very close to 1, and for effective separation the process must be repeated many times.

1. If the separation factor of a process is s, what is the separation factor of an n-fold repetition of this process?

Solution: s^n.

2. The separation factor of a process for the enrichment of uranium 235 is 1.004. How many times must the process be repeated to double the concentration of this element?

Solution: $s^n = 2$

$$(1.004)^n = 2$$

$$n = \frac{\log 2}{\log 1.004}$$

$$= \frac{.3010}{.0017}$$

$$= 177$$

Gaseous diffusion of molecules

The velocity with which molecules of a gas will diffuse through a porous barrier varies inversely as the square root of the molecular weight. If a gaseous mixture of two isotopes with molecular weights m_1 and m_2 ($m_2 > m_1$) starts to diffuse in this manner, the diffused mixture will be enriched in the lighter isotope, the enrichment factor being given by

$$s = \sqrt{\frac{m_2}{m_1}}$$

1. What is the ratio of the rate of gaseous diffusion of helium to that of oxygen? The molecular weights are 8 and 32 respectively.

Solution:

$$\frac{V_n}{V_0} = \sqrt{\frac{32}{8}} = \frac{2}{1}$$

2. In the production of heavy water, hydrogen (molecular weight 2) is separated from its isotope, deuterium (molecular weight 4). If this is to be effected by gaseous diffusion, what is the separation factor?

Solution:

$$s = \sqrt{\frac{2}{1}}$$

$$s = \frac{1.41}{1}$$

3. The separation of uranium 235 from uranium 238 is accomplished by applying gaseous diffusion to these isotopes in uranium hexafluoride, UF_6. Since the atomic weight of fluorine is 19, the molecular weights of the gases are 349 and 352. Find the separation factor of this process.

Solution:

$$s = \sqrt{\frac{352}{349}}$$

$$\log s = \tfrac{1}{2}(\log 352 - \log 348) = .0019$$

$$s = 1.004$$

4. How many times must the process in example 3 be repeated to obtain a 90% concentration of uranium 235? This isotope usually has a concentration of 1/140 in a sample of uranium.

Solution: The required separation factor will be,

$$\frac{9}{10} \div \frac{1}{140} = 126$$

$$(1.004)^n = 126$$

$$n = \frac{\log 126}{\log 1.004} = \frac{2.1004}{.0017}$$

$n = 1250$ times, approximately.

IV. DERIVATION OF FORMULAS

References are so often made to the importance of mathematics in modern science that it seems desirable to give examples of this to our pupils, even if they are necessarily simplified. By means of such examples we may be able to show something of the essential function of mathematical analysis as the physicist's chief weapon in exploring the implications of his experimental data.

1. Assuming equal kinetic energies, ($E = \frac{1}{2}mv^2$), in all the molecules of a gaseous mixture, derive the formula,

$$s = \sqrt{\frac{m_2}{m^1}}$$

for the separation factor by gaseous diffusion.

Solution: Let m_1 and m_2 $(m_2 > m_1)$, be the masses of the two molecules, and v_1 and v_2 their diffusion velocities. Since the kinetic energies are equal, we have,

$$\tfrac{1}{2}m_1v_1^2 = \tfrac{1}{2}m_2v_2^2, \quad \text{or}$$

$$\frac{v^1}{v_2} = \sqrt{\frac{m_2}{m_1}}.$$

The number of molecules that will diffuse through a porous barrier will be proportional to the original concentrations multiplied by these velocities, i.e.,

$$\frac{n^1}{n^2} = \frac{f_1v_1}{f_2v_2} \quad \text{or}$$

$$\frac{n_1}{n^2} \div \frac{f_1}{f^2} = \frac{v_1}{v_2}.$$

The expression on the left is equal to the separation factor. Hence,

$$s = \sqrt{\frac{m_2}{m_1}}.$$

2. Starting with the formula from the theory of relativity for the change in mass of an object referred to a coordinate system with respect to which it is in motion:

$$m_v = \frac{m_0}{\sqrt{1 - \dfrac{v^2}{c^2}}},$$

derive the formula, $E = mc^2$.

Solution:

$$m_v = m_0\left(1 - \frac{v^2}{c^2}\right)^{-1/2}.$$

Expanding by the binomial theorem,

$$m_v = m_0 + \tfrac{1}{2}m_0\left(\frac{v^2}{c^2}\right) + R,$$

where R consists of terms whose denominators contain factors of at least the magnitude c^4, and can therefore be neglected.

$$m_v - m_0 = \frac{\tfrac{1}{2}m_0v^2}{c^2}$$

Since $m_v - m_0$ is equal to the difference in mass, Δm, and $\tfrac{1}{2}m_0v^2$ is the kinetic energy at velocity, v, we have,

$$\Delta m = \frac{E}{c^2}, \quad \text{or}$$

$$E = (\Delta m)v^2.$$

These, then, are just a few of the many problems that can be found. It is hoped that they have shown the great opportunity that exists in atomic physics for providing us with interesting, concrete, and significant applications of mathematics on the high-school level.

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Mathematics bulletin board displays

ROBERT J. OLIVER, *Roseburg High School, Roseburg, Oregon.*

The mathematics bulletin board can be an eye-catching device. Here is a suggestion for those teachers who are becoming pressed for something different.

USE YOUR BULLETIN BOARD! As a teaching aid in mathematics classes, my bulletin board proves both effective and stimulating. Yours can be, too.

Maps, graphs, diagrams, cartoons, pictures of mathematicians, and other selected materials can be arranged attractively on the bulletin board to serve a dual purpose. They will be educational, and they will help in creating a mathematical atmosphere conducive to the study of mathematics.

An interesting variation in the use of the bulletin board may be obtained by displaying geometric figures and solids and posing questions such as: Do you know the formulas for these areas? Can you determine formulas for these volumes?

The materials necessary for constructing such displays are inexpensive and readily available to any teacher. They include heavy white paper, for cutting out letters and for constructing the geometric figures and solids, black paper for background, and miscellaneous articles, such as Scotch tape, scissors, razor blades, a stapler, protractor, compasses, and a straightedge.

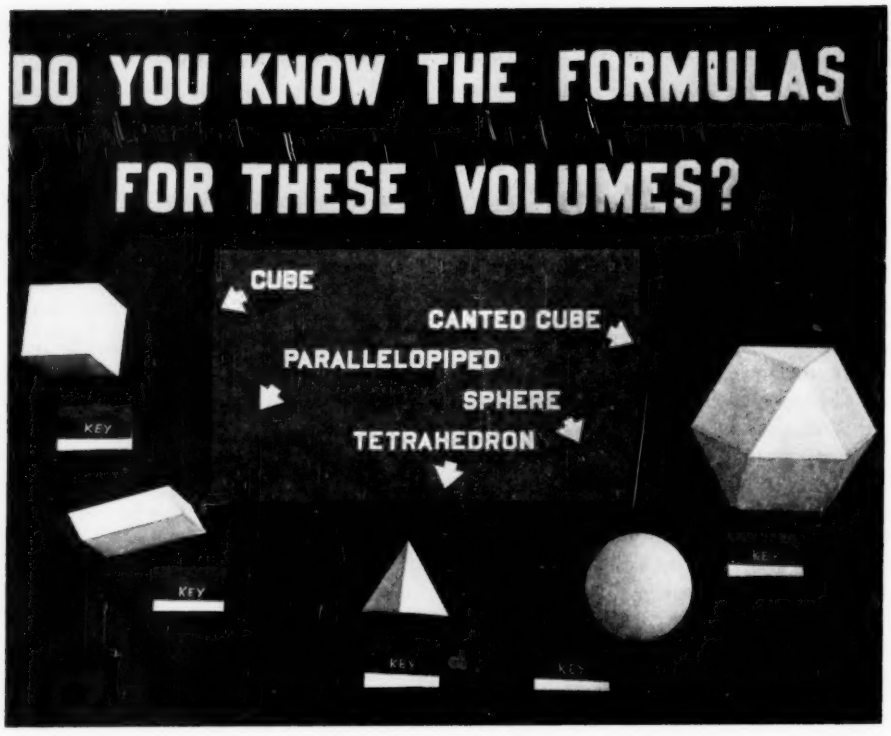
The accompanying illustration shows a three-dimensional bulletin board display, one of a series of displays dealing with areas and volumes. Students help in planning, constructing, and setting up the displays, learning and utilizing many skills in the process.

The keys beneath the solids contain several pages which can be turned up one at a time to reveal hints and suggestions to aid the students in determining the volumes.

The first page of the key for the cube gives the formulas for the volumes of the cube, the right cylinder, and the right prism, and the problem: "Try to determine a *general* formula for finding the volumes of these right solids." If a student is unable to develop the general formula he can turn to the next page of the key on which are listed the formulas for the volumes and the areas of the bases of the volumes. If the student is still unable to develop the general formula he can turn to the next page of the key on which are written the following words: "Remember, a quantity may be substituted for its equal." The lesson, that for right solids volume is equal to the area of the base times the height, is an important key toward finding the formulas for some of the other solids.

Similar keys were developed for each of the remaining solids. On the first page of each key is a lettered diagram of the solid and the words: "Try to determine a formula for this volume." Succeeding pages of each key give further clues and information to aid the student in the development of a formula. Each student strives to use as few of the hints as possible in his development of a formula. After a student has derived a formula he checks with the teacher to be sure that his formula is correct.

Although the displays were intended primarily for geometry classes, some students from all the mathematics classes participated in trying to develop the formulas.



The results of such displays may be witnessed in the attitudes and reactions of the students toward them. The displays are educational, they create interest, they help create a mathematical atmosphere in the classroom, they are easily made, and once made they may be used over and over again.

The editor's mail

In her excellent article "What Is Wrong with School Arithmetic?" (see October '53 issue, p. 407), Gladys Risden claims that "mathematicians" answer the question by saying "not enough drill." What mathematicians? Not this one, or any mathematicians he knows! Most mathematicians I know at the college level say that more *understanding* is needed, more thinking, more emphasis on meaning, and less blind drill, less memory work, less "proof by intimidation." The general opinion among mathematicians is that drill is necessary, but it must be drill in *thinking* as well as manipulation, and it must be accompanied by understanding. Otherwise it is actually harmful. True enough, when all efforts at genuine teaching seem of no avail, it is natural to fall back on "Learn it that way, and do it that way, or else!" but those who make efforts to avoid this last resort will get nothing but support from mathematicians of my acquaintance.

(*signed*) KENNETH O. MAY
Carleton College

The article by Miss Risden in the October '53 MATHEMATICS TEACHER is a very excellent one. It states very clearly what is wrong with school arithmetic, especially at the beginning, and puts the finger on what teachers must do to make it right. It is a pleasure to read such a clear and pointed discussion as Miss Risden has provided.

(*signed*) H. G. WHEAT
West Virginia University

• AIDS TO TEACHING

Edited by Henry W. Syer, Boston University, Boston, Massachusetts and Donovan A. Johnson, University of Minnesota, Minneapolis, Minnesota

BOOKLETS

B-178—Why Study Math

Dept. 2-119 Public Relations, General Electric Company, 1 River Road, Schenectady 5, N. Y.

Booklet; 8½″ by 11″; 8 pages; free.

Content: It is the purpose of this publication to stimulate a greater interest in mathematics. It encourages the high school student to get a good background in mathematics no matter what he plans to do after high school. Mathematics is shown to be essential for the ordinary affairs of everyday life as well as for science, engineering, or industrial designing. A description of how our society has changed is used to emphasize the role mathematics plays in our modern world. The nature of mathematics and its application are described in a simple direct manner. Many occupations are listed in three categories according to the amount of mathematics needed.

Appraisal: The interesting style and appropriate cartoons in red should make this pamphlet easy reading for junior or senior high students. Sometimes its informal treatment of topics such as the nature of mathematics or the influence of training in mathematics may seem superficial and unrealistic. However, the publication should be extremely helpful in telling the high school student the importance of mathematics. The publisher is to be commended for furnishing this needed material at no cost and completely free of advertising.

B-179—Blueprint for Tomorrow

Educational Division, Institute of Life Insurance, 488 Madison Ave., New York 22, N. Y.

Booklet; 8½″ by 11″; 23 pages; with teacher's manual; free.

Description: This is a life insurance unit workbook designed to supplement life insurance chapters in popular textbooks. It is set up in terms of eight lessons. Each lesson contains informational material, charts, exercises, review questions, and an objective test. It covers topics such as kinds of life insurance, cost of insurance, types of insurance benefits and how to plan a life insurance program to meet individual needs. The teacher's manual suggests activities, projects, booklets, charts, and filmstrips to supplement the workbook in addition to supplying a key to all exercises and test items.

Appraisal: This unit will be found to be very easy to use probably because it was prepared with the assistance of three prominent business educators. It presents information in a readable style and gives the student an opportunity to put his knowledge to immediate use through exercises and activities. It provides a free workbook for the student to complete and keep when the unit is finished. There is no advertising of a specific life insurance company or type of insurance. The weaknesses of certain aspects of life insurance will need to be furnished by the teacher or from other sources.

B-180—Consumer Credit Cost Calculator

Household Finance Corporation, 919 North Michigan Ave., Chicago.

Booklet; 6″ by 9″; 6 pages; free.

Description: This publication describes how to compute true interest rates on loans and installment purchases payable in equal installments. The method of computation involves the use of a table based on

cost ratio. The cost ratio is the ratio of the financing cost divided by the total amount paid in installments. A discussion of the meaning of true interest rate is included. Several sample problems are used to illustrate the methods discussed.

FILMSTRIPS

FS. 187 to FS. 193—Basic Numbers: Speed-I-O-Strip

FS. 187—Number Recognition: One and Two Digits

FS. 188—Number Recognition: Three and Four Digits

FS. 189—Number Recognition: Three to Five Digits

FS. 190—Number Recognition: Four to Six Digits

FS. 191—Number Recognition: Five and Six Digits

FS. 192—Number Recognition: Seven to Nine Digits

FS. 193—Number Recognition: Six Tests in Reading Numbers

Society for Visual Education, 1345 Diversey Place, Chicago 14, Ill.

B & W, 35 mm.

Description: These seven filmstrips have a single number on each frame with the number of digits in the title. The spacing has some variety. They are intended to be used in tachistoscopic presentation to aid in acquiring speed in number recognition. There are forty frames of numbers on each filmstrip.

In each of the following descriptions the number of frames will mean those showing numbers. In general there are five more frames of titles and description.

FS. 194—Addition: Fifteen Easiest Combinations (30 frames)

FS. 195—Addition: Thirty Easy Combinations (60 frames)

FS. 196—Addition: Nineteen Zero Combinations (38 frames)

FS. 197—Addition: Thirty-six More Combinations (72 frames)

FS. 198—Subtraction: Fifteen Easiest Combinations (30 frames)

FS. 199—Subtraction: Thirty Easy Combinations (60 frames)

FS. 200—Subtraction: Nineteen Zero Combinations (38 frames)

FS. 201—Subtraction: Thirty-six More Combinations (72 frames)

FS. 202—Multiplication: Group I (50 frames)

FS. 203—Multiplication: Group II (50 frames)

FS. 204—Multiplication: Group III (50 frames)

FS. 205—Multiplication: Group IV (50 frames)

FS. 206—Division: Group I (46 frames)

FS. 207—Division: Group II (44 frames)

FS. 208—Division: Group III (44 frames)

FS. 209—Division: Group IV (46 frames)

Description: These sixteen filmstrips have problems, each with two digits to fit the description of the title, arranged vertically without any sign of operation. There is a line under each problem. A frame with the problem without an answer is followed by a frame with the problem in the same position, but the answer is present as well. These are used for tachistoscopic presentation also.

Appraisal: It is hard to avoid the implication that a purely drill theory of learning combinations is behind the design of these filmstrips. One must answer for himself how important it is to stress the actual speed at which one can perceive the problem to be solved. Even increasing the speed of perception and response for these number combinations is but a small part of the learning. Such considerations are inherent in the use of the tachistoscope for the teaching of number combinations.

If these fit in with your philosophy, they will be very useful.

FS. 210—How to Tell Time. Part I, The Hour and Half-hour; Part II, The Minutes

Popular Science Publishing Company, Audio-Visual Division, 353 Fourth Avenue, New York 10, New York.

B & W; 35mm; 45 frames each; $2.50 each.

Description: A cardboard clock is used by Ann to show seven-year-old Rodger the function of the hour and minute hands and the direction the hands move. Practice is given in telling time by hours and half-hours in Part I and by minutes in Part II. Various clock settings are shown with questions to be answered by the viewers. After illustrating the times of Rodger's daily activities, the last six frames, designed to be projected on the blackboard, show a clock face without hands. The children may then draw the hands of the clock to represent the stated time.

Appraisal: The filmstrip uses good technique in frequently inviting audience response and participation. The use of cardboard clocks by pupils would probably be more effective than viewing this filmstrip, provided the teacher plans her presentation as completely as the film has been organized.

FS. 211—International Date Line

Popular Science Publishing Company, 353 Fourth Avenue, New York 10, New York.

B & W; 35 mm; 37 frames; $2.50.

Description: Trans-Atlantic flights illustrate the difference in clock hours between east and west flights although flying time may be the same. A 24-hour flight around the world in 24 hours demonstrates the need for a specific line such as the International Date Line to designate the beginning of a given date. The change in time as one travels from one time belt of the United States to another is illustrated with maps and clocks. The filmstrip ends with questions to stimulate discussion.

Appraisal: Although this filmstrip consists primarily of maps it will furnish a convenient sequence which emphasizes the need for time zones and a date line.

FS. 212 to 227—Using Numbers

FS. 212—Counting to 5—Lesson 1

FS. 213—Counting to 10—Lesson 2

FS. 214—Reading Numbers to 10—Lesson 3

FS. 215—Counting Numbers to 10—Lesson 4

FS. 216—Counting by 10's to 30—Lesson 5

FS. 217—Counting by 10's to 50—Lesson 6

FS. 218—Counting by 10's to 80—Lesson 7

FS. 219—Counting by 10's to 100—Lesson 8

FS. 220—Counting from 10 to 15—Lesson 9

FS. 221—Counting from 15 to 20—Lesson 10

FS. 222—Counting from 20 to 40—Lesson 11

FS. 223—Counting from 40 to 100—Lesson 12

FS. 224—Reading Numbers to 50—Lesson 13

FS. 225—Reading Numbers to 100—Lesson 14

FS. 226—Working with Numbers to 100—Lesson 15

FS. 227—Writing Numbers to 100—Lesson 16

Encyclopaedia Britannica Films, Inc., 1125 Central Ave., Wilmette, Illinois.

35 mm. filmstrips; Black and white: John Rand Clark and Caroline H. Clark, Educational collaborators, 1948, $3.00 each or $43.20 for the entire set of 16 filmstrips.

Description of FS. 212 to FS. 227: Each of these filmstrips is a separate lesson. Each consists of two parts, one part for the teacher and the other for the pupils. The first frames of each film give instructions to the teacher. These instructions state the objectives of the lesson, how to utilize the film effectively, and how to attain maximum learning from the films. The pictures for the pupils show how to count items, such as coins, stamps, blocks, dolls, dominoes, apples, balls. These objects are grouped to illustrate the meaning of numbers and the addition of one or the subtraction of one. Numbers to be read or written are numbers as used for clocks, calendars, page numbers, game scores, or groups of objects. Each frame has a statement, question, directions, or a problem to be read by the teacher. These captions usually require a response from the viewer. The first frames review the previous lesson and the final frames are devoted to summary questions.

Appraisal of FS. 212 to FS. 227: These filmstrips use clear cut pictures of objects to illustrate concretely the size of numbers. Throughout the series the emphasis is on the meaning of numbers and the comparison of numbers. For example, place value is emphasized by learning to count by tens before learning to count from 10 to 15. Frequently, the value of a number is related to the number of tens and units it contains. New concepts are developed in an organized sequence that follows good learning principles. This series also recognizes the importance of repetition, review, and summary. Most of the frames have attractive pictures and appropriate captions. As in most arithmetic filmstrips, this series will suggest to the teacher a variety of ways to build number concepts using actual objects rather than pictures. Considerable verbal material is included but its purpose is to help the teacher get the viewer to react to the learning situation.

Applications editor resigns

Sheldon S. Myers, editor of the Applications department of THE MATHEMATICS TEACHER, has resigned. Sheldon has found it necessary to devote his time to his new position at Ashville-Harrison High School, Ashville, Ohio and to the "felt need" for a doctor's degree from Ohio State University.

Readers of THE MATHEMATICS TEACHER will be sorry to have Sheldon give up his responsibilities for this department. He has made an excellent contribution to the magazine. We all wish Sheldon Myers good luck while at the same time threatening to call on him again when he has that desired degree.

The Purdue-General Electric Fellowship Program

The Purdue-General Electric Fellowship Program for Secondary School Teachers of Mathematics has been renewed for 1954. There will be fifty Fellowships available for high school teachers of mathematics in the eleven-state area including Illinois, Indiana, Iowa, Kentucky, Michigan, Minnesota, Missouri, Ohio, Tennessee, West Virginia and Wisconsin.

The Fellowships cover essentially all expenses, including cost of travel both ways between the Fellow's school and Lafayette, lodging and meals in one of the University dormitories for the period of the program, June 21–July 31, 1954, and course fees. The program includes three courses, carrying a total graduate credit of eight hours, and a seminar on problems of teaching and counselling in high school. Special features include inspection trips to plants of the General Electric Company in Fort Wayne, Indiana, and Danville, Illinois.

Further information and application forms for the Fellowships will be sent upon request addressed to Professor Ralph Hull, Head, Department of Mathematics, Purdue University, Lafayette, Indiana.

• APPLICATIONS

Lyman C. Peck, Iowa State Teachers College, Cedar Falls, Iowa, guest editor of this column, discusses gears, gear teeth, and involutes of circles. He shows how gears are designed on the basis of an involute of a circle and how involutes can be made significant in a high-school class.

Gears, gear teeth, and mathematics

Gears of clocks, watches, bicycles, and automobiles are constantly behind the scenes in our environment. Many youngsters are fascinated by the insides of a timepiece or of the differential of a car. To a certain extent, then, gears and gear wheels have universal appeal.

GEARS AND PULLEYS

It is interesting to see how the action of meshed gear wheels is analogous to the operation of a pair of pulleys transmitting motion from one to the other through a crossed belt. The point of intersection of the crossed belt divides the center-to-center distance of the pulleys into segments whose ratio is the same as the ratio of the corresponding pulley diameters. Figure 1 illustrates this principle. Since the number of teeth on a gear wheel is directly proportional to its diameter, we see that the *pitch point* of a gear pair (equivalent to the point of intersection for a pulley belt) has its location on the center-to-center line determined by the ratio of the number of teeth in the gears.

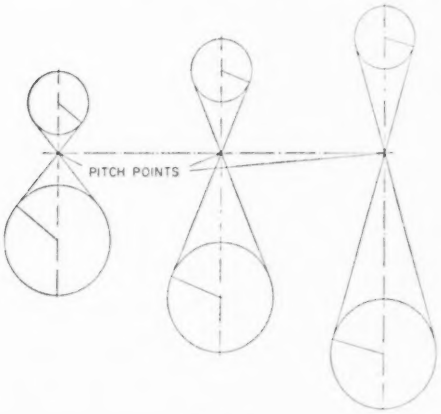


Figure 1

The gears usually found in inexpensive modern clocks and in wind-up toys are constructed according to the above principle. However, unless great care is exercised in shaping the teeth, the transmission of motion from one gear to the other is not uniform. The rattling buzz heard when the wheels of a toy automobile are allowed to run free is evidence of this. Consequently, gears which must bear heavy loads or work at extremely high speeds must have a very particular type of construction.

In terms of the geometry of gear construction, the requirement of transmitting motion uniformly means that the common normals at the point where teeth engage should be tangent to the base circles of the gears. When this principle is analyzed mathematically, it is found that the shape for the gear teeth must be that of the involute of a circle.

The involute of a circle is a curve which, traditionally, no student of mathematics meets before the section on parametric equations in his college analytic geometry. Often he meets it for the first time in calculus. However, setting aside the algebraic development of its equation, the in-

volute of a circle is readily within the grasp of high school students.

THE INVOLUTE OF A CIRCLE

Figure 2 shows the involute of a circle, and suggests how it can be made by unwrapping a string from around a circle. Note that any normal to the involute is tangent to the base circle. It is not very difficult to make a model out of wood, plexiglass, or aluminum which would illustrate these principles. Figure 3 gives helpful suggestions toward that end. Notice in Figure 3 that the common normals to the involute teeth pass through the pitch point, and further that they follow the direction which would be taken by a pulley belt passing around the base circles. In a sense, then, a gear pair with involute teeth is equivalent to a crossed belt and a pair of pulleys.

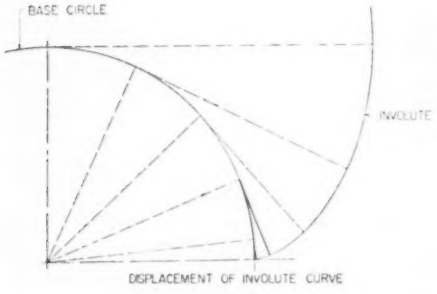


Figure 2

The manner by which commercial gear cutting machines make gears with involute teeth is strikingly similar to the process pictured in Figure 3. The gear, placed on a larger circle, is rolled into contact with the generating tool. As this motion is continued, the involute tooth is made, as in Figure 4. The commercial success of this type of gear tooth is directly due to the simplicity of this process.

Some of the most useful concepts and operations which science and industry borrow from mathematics are intrinsically simple. Thus it is with the involute curve as applied to gear teeth.

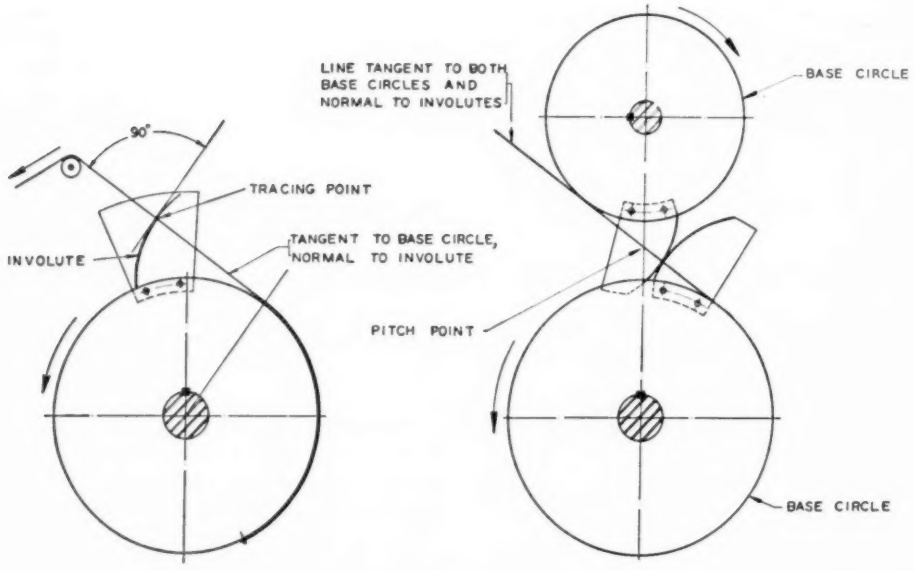


Figure 3

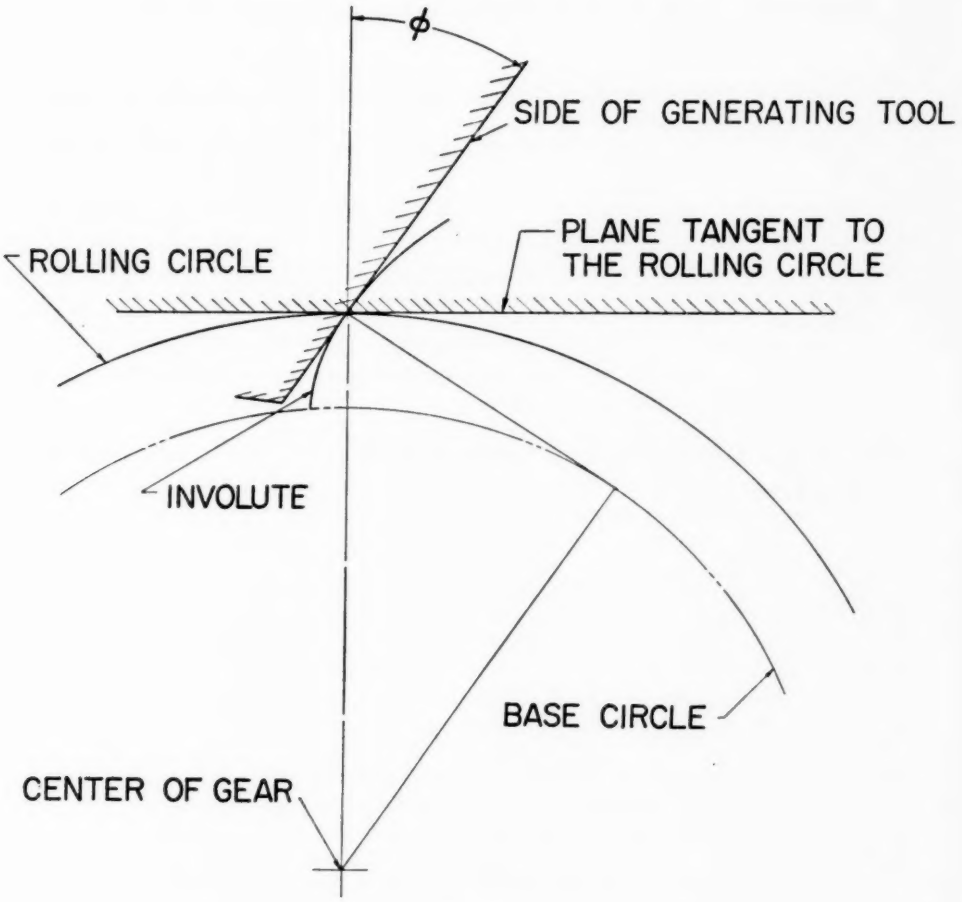


Figure 4

Acknowledgment: The facts and ideas in this article were obtained from Griffith, Bain, "Gear Wheels and Applied Mathematics," *General Motors Engineering Journal*, Vol. 1, No. 2, September-October 1953, pp. 33–39. All diagrams were reproduced from this publication with the kind permission of the Chevrolet Motor Division.

Back issues—?

One of the book section editors is very anxious to fill in some back files of THE MATHEMATICS TEACHER. Any reader who is willing to sell any of the issues listed please notify Professor C. B. Read, University of Wichita, Wichita 14, Kansas.

1929—January, February, and March
1926—All issues except February and March
1925—February, November, and December
1921—January and December
1918—June and December
1917—September

• DEVICES FOR A MATHEMATICS CLASSROOM

Edited by Emil J. Berger, Monroe High School, St. Paul, Minnesota.

Nona Mary Allard, Alexander Ramsey High School, St. Paul, Minnesota, presents her solution to the time-consuming problem of collecting aids to help the pupil experience the thrill of discovery in mathematics. The editor, Emil J. Berger, suggests a device to facilitate the teaching of infinite series in advanced courses in the high school. Mr. Berger also suggests problem material for solid geometry.

An individual laboratory kit for the mathematics student

Contributed by Nona Mary Allard

The thrill of discovery is a powerful incentive to students of physics and chemistry. Mathematics students, too, can be motivated by discovery experiences. However, the collection, care, and distribution of the materials needed to employ the discovery method can be a prohibitively time-consuming chore for the teacher.

One solution to this problem is to assemble with each student his own personal mathematics laboratory kit. Such a kit (as the one which will presently be described) is useful for developing, demonstrating, and illustrating a great variety of mathematical facts, and is, as well, inexpensive, convenient, and easy to store.

If individual drawer space is not available for all students, shoe boxes or any other readily obtainable set of boxes of uniform size may be used to serve as containers. Each such kit should contain such obvious tools as a ruler, compasses, protractor, scissors, and pencil, and also the less obvious but immensely handy paper punch. Other materials that should be included are: a small bottle of paste or glue, map pins, push pins, a piece of corkboard or celotex large enough to serve as a mounting for graph paper, elastic thread, string, paper fasteners, a quantity of pick-up sticks, and a razor blade. Expendable materials such as squared cross-section paper, construction paper, and four-ply cardboard strips $\frac{1}{2}''$ wide may be distributed as needed.

With the aid of such simple equipment and materials as these, a great many valuable devices can be prepared. Two such devices that might be suggested are the angle bisector and angle trisector.

The angle bisector device is a triangle constructed of three cardboard strips held together with paper fasteners inserted through holes made with the paper punch (Figure 1). Sides AC and AB must be

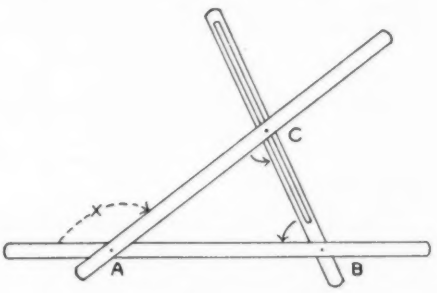


Figure 1

equal in length from center to center. Side BC is fastened to side AB at B with a single paper fastener. Vertex C is fixed in a long slot made with the paper punch. By moving C along this slot the size of the exterior angle at A may be varied as desired.

To bisect a given angle X, place vertex A of the device on the vertex of the given angle so that AC of the device coincides with one side of angle X and the extension of AB through A coincides with the other side of angle X. Since the interior angles at B and C are the equal base angles of an isosceles triangle and their sum is equal to the exterior angle at A, either one of them will be equal to one-half of angle X for all values of X less than 180°.

The angle trisector device is made of two cardboard strips glued or stapled together at right angles to form a T-square with arms of equal length (t). Suppose angle BAC is the angle to be trisected. Draw a line parallel to AC at a distance of t units from it (Figure 2). Now if the T-square is placed in the position indicated in the diagram, then it can be shown that angle BAY = angle YAZ = angle ZAC.

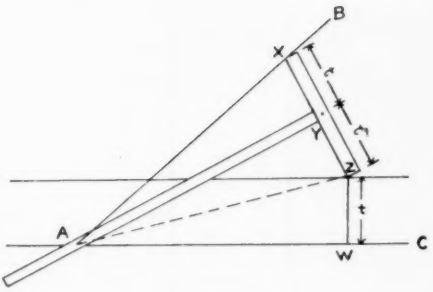


Figure 2

With the aid of the auxiliary line AZ it is easily proved that the three right triangles XAY, YAZ, and ZAW are congruent, and that the three angles named are corresponding parts of these triangles. Hence angle BAC is trisected.

These two devices are by no means all that can be made from the tools and materials included with the student's individual mathematics laboratory kit proposed in this article. Future issues of THE MATHEMATICS TEACHER will contain descriptions of other models, devices, and experiments.

A model for teaching infinite series to high-school students

by Emil J. Berger

An interesting special topic which many high school teachers introduce in their second year algebra is infinite series. Unfortunately, however, the topic is rarely developed in sufficient detail to give students any real insight into the fundamental ideas which are related to the topic. Usually work with infinite series in high school classes is limited to consideration of elementary numerical manipulations with simple arithmetic and geometric progressions. However, it would seem proper that, if an introduction to this important topic is to be worthwhile for the student, then attention should be given to the task of helping him develop a respectable understanding of such related ideas as *sequence of terms, limit of a sum, sum of a series, convergent series, divergent series,* and *law of formation.*

Numerical examples of the type referred to above are fine, but at the level of maturity of high school students such illustrations are not concrete enough to communicate the meanings of the ideas involved.

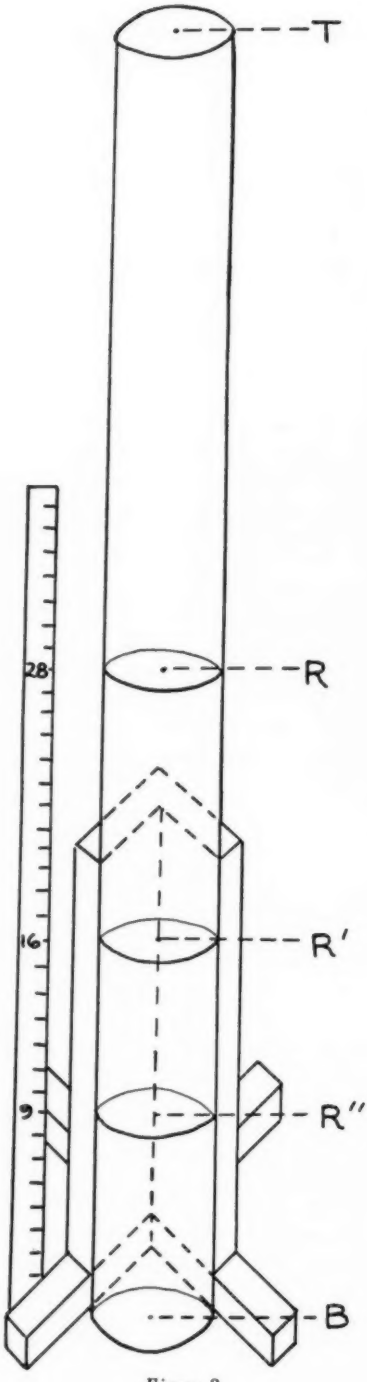


Figure 3

Now it would be wonderful if at this point we could state that there is a teaching aid which may be called upon to resolve this little problem of communication, but unfortunately this is not the case. However, we will offer a suggestion for a device which we hope will prove to be of some assistance in presenting the ideas involved.

The device we propose was inspired by an exercise from Walter W. Hart's book, *Essentials of Algebra, Complete Second Year Course.* We re-state the problem so that the reader may see how the building of the device got underway.

> Suppose a rubber ball is dropped from a window that is 40 feet above ground; that it rebounds 20 feet and again drops; that it rebounds 5 feet and drops; and so on for a *very large number* of times. How far does the ball travel?[1]

Upon reading this problem several students in the department editor's class wondered whether this was an actual physical situation that could be reproduced, or whether the numbers given had been so chosen as to provide a semi-realistic problem in infinite geometric progressions. They decided to build a device, and for this purpose obtained the following materials: one piece of sheet acetate $12'' \times 60'' \times .005''$, one piece of wood $\frac{7}{8}'' \times 2'' \times 18''$, one piece of wood $\frac{7}{8}'' \times 2\frac{7}{8}'' \times 18''$, two pieces of wood each $\frac{7}{8}'' \times \frac{7}{8}'' \times 8''$, one new golf ball, several new ping pong balls, several small rubber balls ($1\frac{1}{2}''$ or less in diameter), a quantity of natural-colored rubber binders, and four or five red rubber binders.

The device illustrated in Figure 3 is similar to the one which the boys constructed. It consists of an upright stand and a closed cylindrical surface. The upright stand is in two pieces which are joined together as illustrated. One of these pieces is in the form of a right angled trough whose inside dimensions are $2'' \times 2'' \times 18''$ (Fig. 4). The other part is in the form of a cross made from two pieces of wood which are notched and lap joined at

[1] Used by special permission of the publishers, D. C. Heath and Company, Boston, Mass.

their midpoints (Fig. 5). When nailed together the cross-shaped part and the trough form a support for holding the closed cylindrical surface in an upright position. Care must be exercised to insure that this stand is plumb. The closed cylindrical surface is nothing more than a roll of sheet acetate, rolled so that its diameter is $2\frac{1}{2}''$. Five natural-colored rubber binders slipped over the ends and distributed along its length help maintain its shape. Two more natural-colored rubber binders slipped over both the trough and the cylindrical surface provide sufficient support to hold the cylindrical surface in vertical position.

The reader will note that the construction described leaves the cylindrical surface open at both ends. This is a necessary feature of the device, and reference will be made to this fact again later. The yardstick which appears in Figure 3 is not part of the device. The black rings which appear on the hollow cylinder are red rubber binders and represent the first three terms of some infinite series. Their function will be explained presently.

When the boys tested their device by dropping different balls into the hollow cylinder they found that when a new golf ball was dropped on a marble slab the results supposed in the problem quoted above were very nearly duplicated. In fact, they obtained the figures $60''$, $31''$, and $16''$ for the initial drop and the first two rebounds. However, the results with the golf ball were far from typical. For other balls dropped on different materials they found different ratios.[2]

To show how the ideas related to infinite series listed earlier in this article are illustrated by means of this device, we present the results of an actual classroom experiment in which a ping pong ball was dropped from the $60''$ level at T through the tube onto a desk-top made of cork-base linoleum. Following the first rebound at B the ball rose to R, a height of $28''$; this value may be thought of as being the first term in the sequence of rebounds. Following the second bounce the ball rose to R', a height of $16''$; and following the third bounce the ball rose to R'', a height of $9''$. The last two values given, namely $16''$ and $9''$, may be thought of as representing the second and third terms respectively in the sequence of rebounds.

Figure 4

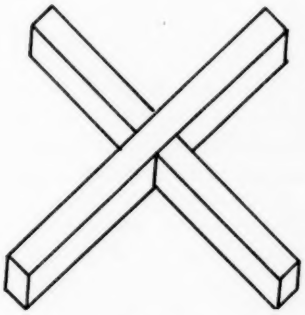


Figure 5

[2] These facts are in accordance with the physical principles involved. Newton discovered that for freely falling bodies the coefficient of restitution is constant and equal to $\sqrt{h_2/h_1}$, where h_1 and h_2 are heights attained after successive rebounds, and that this constant depends on the particular substances that come in contact.

For the benefit of those readers who may wish to repeat this experiment for themselves it should be explained that the heights R, R', and R'' of successive rebounds were not all determined in the same set of successive bounces. First the ball was dropped and R was determined, then the ball was dropped a second time and R' was determined. To determine R'' the ball was dropped a third time. As the various heights were determined red rubber binders were moved into position along the cylinder. Finally, the heights of the positions, R, R', and R'', were measured with a yardstick as shown in Figure 3.

Inspection of the first three terms of the sequence of rebounds, $28''$, $16''$, $9''$, $\cdots$, does not reveal at once that these terms approximate a geometric progression, but suppose we find the ratio of the second term to the first term; it is

$$\frac{16}{28}=\frac{4}{7}.$$

Now since $\frac{4}{7}\times 16''=9\frac{1}{7}''$ is a good approximation for the height of the third term as actually obtained, it would certainly not be unreasonable for one to interpret this sequence of rebounds as a geometric progression whose law of formation, at least in an approximate sense, is satisfied by the expression $28\cdot(\frac{4}{7})^{n-1}$, where n is the order of the bounce.

One might ask at this point why wasn't the $60''$ drop taken as the first term in a sequence of *drops?* Although this is partially conjecture, it would appear that the ping pong ball reaches its terminal velocity some place along the initial $60''$ drop and so the $60''$ term cannot be thought of as part of a sequence of drops. If one knew the critical height, or went to the trouble to find it, then this height could certainly be considered as the first term in the sequence of drops, but this is another experiment and our device is not adaptable for conducting it. It is because of the foregoing explanation that the ping pong ball experiment rather than the one with the golf ball was selected for this illustration.

If we ask the question, how far does the ball travel, we are asking for the sum of the initial drop and the series, $56''+32''+18''+\cdots$, and we must consider the sum S of the series whose general term or law of formation is $56\cdot(\frac{4}{7})^{n-1}$, where n is the order of the bounce as before. The mathematical facts so far displayed would prompt one to agree that the series in question is indeed an infinite series and that to find its sum S one must consider the limit of the sum of n terms as $n\to\infty$. In other words when we ask for the sum S of the series we really want

$$\frac{\lim 56(1-[4/7]^n)}{n\to\infty \quad 1-4/7}.$$

Since the bouncing ball appears to come to rest eventually no student will doubt that the distance the ball travels following the initial bounce is approximately the limit in question, and that the sum S of the series converges to this limit.

Note that we state "the bouncing ball appears to come to rest." If the desk-top and ping pong ball were perfectly elastic substances, then the ball would continue indefinitely to rebound to a height of $60''$ once it was released from that height. The series in this case would have the form $120''+120''+120''+\cdots$, which any high school student will agree approaches no limit. Of course no two substances are perfectly elastic, but the description of such an ideal situation appeals to students of high school age, and in this way helps

Anyone who has a learning aid which he would like to share with fellow teachers is invited to send this department a description and drawing for publication. If that seems too time-consuming, simply pack up the device and mail it. We will be glad to originate the necessary drawings and write an appropriate description. All devices submitted will be returned as soon as possible. Send all communications to Emil J. Berger, Monroe High School, St. Paul, Minnesota.

communicate what is meant by an infinite series that is divergent.

Because the observations reported above for the heights of the various bounces are not in precise geometric ratio, one might argue that this device is not particularly useful. It should be remembered, however, that anyone who uses a device like this must be willing to agree that while closeness to the real law of formation may be desirable, actual observations are relatively unimportant except that they serve as a handy vehicle for presenting the theory involved. Moreover, most students are willing to accept the discrepancies which arise as errors in measurements.

Acknowledgment: The department editor wishes to express his gratitude to Professor Harold Hughesdon of St. Thomas College for the advice and timely assistance received from him in the preparation of this article.

"Eureka"

by Emil J. Berger

There are times when just looking at an object seems to "turn" it into a learning aid. As a former classmate of the department editor once put it—"It isn't what one knows, it's what one sees!"

Recently while this editor was browsing through a sporting goods store in search of original problem material for a projected test in solid geometry the brightly colored, right circular, cylindrical, metal containers in which sporting goods companies dispense tennis balls (three balls to a container) took on a new "hue" of meaning. Three spheres inscribed in a circular cylinder should certainly be a model for something—but what? Then the idea hatched. Why of course! The three balls and their container are an honest-to-goodness illustration of the theorem which states that the ratio of the surface of a sphere to the lateral area of its circumscribed cylinder is unity.

To prove this theorem one needs only recall that the area of a sphere is πd^2, and that the lateral area of the circumscribed cylinder is πdh where $d = h$.

In the case of our newly acquired model, the altitude of the cylinder is $3d$, but there are three balls, so the ratio is unity as before.

Second Mathematics Institute at Rutgers University

The Association of Mathematics Teachers of New Jersey and of Rutgers University are jointly sponsoring a Second Mathematics Institute to be held at Rutgers University, New Brunswick, New Jersey, July, 1954.

The Institute will provide an opportunity for teachers of elementary, junior high school, and senior high school mathematics to study the problems of mathematics for a period of ten days while living and working together as a group.

Study groups, laboratories, outstanding resource specialists, planned social program. 2 points of academic credit granted by the Rutgers School of Education

Tuition fees for residents of New Jersey..... $11.00 credit hour

Tuition fees for non-residents—additional..... $ 3.00 credit hour

Registration fee..... 5.00

Total fee without academic credit..... 20.00

Dormitory (double occupancy)—approximately. 10.00

Registration—Wednesday afternoon, July 7, 1954

Sessions—Thursday, July 8 through Friday, July 16.

Brochure upon application to

Director of Summer Session
Rutgers University
The State University of New Jersey
New Brunswick, New Jersey

Edited by Phillip S. Jones, University of Michigan, Ann Arbor, Michigan.

A very interesting account of the development of the idea of complex numbers. The story as here told sheds light on the role of intellectual curiosity in the development of mathematics.

Complex numbers: an example of recurring themes in the development of mathematics—I

by Phillip S. Jones

The story of the development of the concept and uses of complex numbers provides a fine example of how the history of mathematics may shed light on the meaning of terminology, the relative roles of "practical" needs and intellectual curiosity in the motivation of mathematicians, the utility of pure mathematics, and the development not only of mathematics itself but also of the concepts of rigor and proof. The story also involves intrigue and illustrates the international nature of mathematical scholarship. Hence, properly told, this story does much more than merely provide interest and motivation; in any event, it surely does no less.

Some historians[1] trace, in a negative sort of way, the beginnings of the concept of an imaginary number to Heron's *Stereometria* (*circa* 75 A.D.). In this Heron states a problem about a pyramid which properly computed would have led to an imaginary number for the length of a line. Heron (or a later copyist) reversed the order of subtraction, and thereby missed both the imaginary number and the correct answer. To the writer this hardly seems to represent a phase in the development of complex numbers, but it does illustrate that in Heron's day, as in our own textbooks, not all apparently "real" problems were actually based upon real-life situations or measurements. Further this illustrates a situation which was later emphasized by René Descartes, that imaginary numbers or roots may be the algebraic counterpart of non-existent intersections in geometry, or of impossible physical conditions. In these situations the imaginary numbers obtained may however tell a real story since their mere occurrence may be significant in itself.

Still negative in its nature, but of real significance in the story of the growth of the idea because of its explicit recognition of the problem, is the statement of the Hindu Mahavir (*circa* 850), "as, in the nature of things, a negative is not a square, it has no square root."[2]

The first printed book to contain algebra, Luca Pacioli's *Summa de Arithmetica Geometria* (Venice: 1494), in discussing quadratic equations, explained in words that a solution was possible only if the constant term was less than or equal to the square of one-half the coefficient of the

[1] D. E. Smith, *History of Mathematics* (Boston: Ginn and Co., 1925), p. 261. Smith also gives several references to other discussions of Heron's problem.

[2] M. Rangacarya, *The Ganita-Sara-Sangraha of Mahaviracarya with English Translation and Notes* (Madras: 1912), p. 7 of the English translation. The sentence before the one quoted above was, "The square of a positive as well as of a negative (quantity) is positive; and the square roots of those (square quantities) are positive and negative in order."

HIERONYMI CARDANI MEDIOLANENSIS, CIVISQVE BONONIENSIS, PHILOSOPHI, MEDICI ET Mathematici clariſsimi,

OPVS NOVVM DE PROPORTIONIBVS NVMERORVM, MOTVVM, PONDERVM, SONORVM, ALIARVMQVE RERVM

menſurandarum, non ſolùm Geometrico more ſtabilitum, ſed etiam uariis experimentis & obſeruationibus rerum in natura, ſolerti demonſtratione illuſtratum, ad multiplices uſus accommodatum, & in V libros digeſtum.

PRAETEREA

ARTIS MAGNÆ, SIVE DE REGVLIS ALGEBRAICIS, LIBER VNVS, ABSTRVSISSIMVS

& inexhauſtus plane totius Arithmeticæ theſaurus, ab authore recens multis in locis recognitus & auctus.

ITEM

DE ALIZA REGVLA LIBER, HOC EST, ALGEBRAICAE logiſticæ ſuæ, numeros recondita numerandi ſubtilitate, ſecundum Geometricas quantitates inquirentis, neceſſaria Coronis, nunc demum in lucem edita.

Opus Phyſicis & Mathematicis imprimis utile & neceſſarium.

Cum Cæſ. Maieſt. Gratia & Priuilegio.

BASILEÆ.

Figure 1

first degree term (the coefficient of the second degree term being unity).

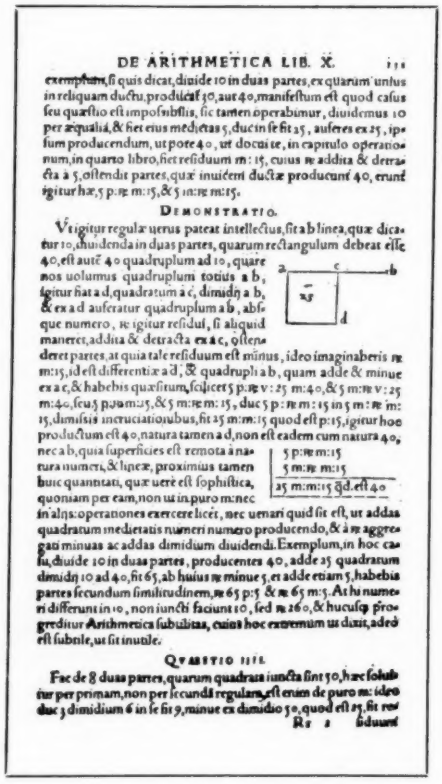
The real beginning of complex numbers is to be found in the work of the Italian, Jerome Cardan. His *Ars Magna* was first printed in Nuremberg in 1545. Our Figure 1 showing the title page of its 1570 edition printed in Basel suggests many other things, however. Not only does it suggest the internationalism of printing revealed by the sources of these two editions, but also it points out the cosmopolitan nature of Cardan who is here titled *Mediolanensis* (of Milan) as well as *civisque Bononiensis* (citizen of Bologna). This recalls the heyday of the city-states of Italy, and the fact that Cardan was a contemporary of Machiavelli (1469–1527). This latter fact suggests that the famous "steal" of the solution of the general cubic by Cardan from Nicholas of Brescia (Tartaglia), using as a ruse the story of a noble sponsor, a possible patron, who was interested in Tartaglia's discovery, was quite in keeping with the political morals portrayed in Machiavelli's *The Prince*, as well as displaying something of the relationships between the persons of wealth, the nobility, and the scholars of the day.

Of further interest on the title page is the characterization of Cardan as *philosophi, medici, et mathematici clarissimi.* For not only was Cardan himself a remarkable person, famous in the history of medicine, cryptography, and gambling,[3] but also this suggests the parallel growth of philosophy and science with mathematics, and the continuing relationships which exist today among these disciplines.

[3] There have been several biographies of Cardan. The most recent is by Oystein Öre, *The Gambling Scholar* (Princeton University Press, 1953). Cardan's own autobiography has been translated as *The Book of My Life* (New York: 1930, translated by Jean Stover; and London: J. M. Dent and Son, 1931), while his medical achievements are emphasized in Henry Morley's *Jerome Cardan: The Life of Girolamo Cardano of Milan, Physician* (London: Chapman and Hall, 1854). In addition to W. G. Walters, *Jerome Cardan a Biographical Study*, (London: 1898) there have been many journal articles about him and his work as well as short biographies in books and in *Portraits of Eminent Mathematicians, Portfolio II* published by *Scripta Mathematica.*

Lastly, the words *Ars Magnae, sive de Regulis Algebraicis* followed by a reference to *Arithmeticae* in contrast to which algebra was the "great art," with the added comment *"fecundum geometricas quantitates inquirentis"* (fruitful in inquiries concerning geometric quantities) shows that the interrelatedness of these three parts of mathematics was recognized then as it should be emphasized in our teaching now.

Figure 2 shows page 131 of the book whose title page we have just discussed. The first two lines state Cardan's famous problem "divide 10 into two parts such that the product of one times the remainder is (30 or) 40." Cardan immediately says *manifestum est—impossibilis.* He does not explain why this is mani-



DE ARITHMETICA LIB. X. 131

exemplum, si quis dicat, diuide 10 in duas partes, ex quarum unius in reliquam ductu, prodeat 30, aut 40, manifestum est quod casus seu quæstio est impossibilis, sic tamen operabimur, diuidemus 10 per æqualia, & fiet eius medietas 5, ducin se fit 25, auferes ex 25, ipsum producendum, ut pote 40, ut docui te, in capitulo operationum, in quarto libro, fiet residuum m: 15, cuius R addita & detracta a 5, ostendit partes, quæ inuicem ductæ producunt 40, erunt igitur hæ 5 p:R m:15, & 5 m:R m:15.

DEMONSTRATIO.

Vt igitur regulæ uerus pateat intellectus, sit a b linea, quæ dicatur 10, diuidenda in duas partes, quarum rectangulum debeat esse 40, est autē 40 quadruplum ad 10, quare nos uolumus quadruplum totius a b, igitur fiat a d, quadratum a c, dimidij a b, & ex a d auferatur quadruplum a b, absque numero, R igitur residui, si aliquid maneret, addita & detracta ex a c, ostenderet partes, at quia tale residuum est minus, ideo imaginaberis R m:15, id est differentiæ a d, & quadrupli a b, quam adde & minue ex a c, & habebis quæsitum, soluetes 5 p:R v:25 m:40, & 5 m:R v:25 m:40, seu 5 p:R m:15, & 5 m:R m:15, duc 5 p:R m:15 in 5 m:R m:15, dimissis intricationibus, fit 25 m:m:15 quod est p:15, igitur hoc productum est 40, natura tamen a d, non est eadem cum natura 40, nec a b, quia superficies est remota à natura numeri, & lineæ, proximius tamen huic quantitati, quæ uere est sophistica, quoniam per eam, non ut in puro m: nec

5 p:R m:15
5 m:R m:15
25 m:m:15 q̄d. est 40

in alijs operationes exercere licet, nec uerari quid sit est, ut addas quadratum medietatis numeri numero producendo, & à R aggregati minuas ac addas dimidium duidendi. Exemplum, in hoc casu, diuide 10 in duas partes, producentes 40, adde 25 quadratum dimidij 10 ad 40, fit 65, ab huius R minue 5, et adde etiam 5, habebis partes secundum similitudinem, R 65 p:5 & R 65 m:5. At hi numeri differunt in 10, non iuncti faciunt 10, sed R 260, & hucusq; progreditur Arithmetica subtilitas, cuius hoc extremum ut dixit, adeò est subtile, ut sit inutile.

QVAESTIO XIII.

Fac de 8 duas partes, quarum quadrata iuncta sint 50, hæc solubitur per primam, non per secundā regularem, est enim de puro m: ideo duc 3 dimidium 6 in se fit 9, minue ex dimidio 50, quod est 25, fit res

Rx 2 diduint

Figure 2

festly impossible, but our students can be led to do good "functional" thinking by suggesting that they consider the products of pairs of numbers whose sum is 10, or, on a higher level, that they graph $y=x(10-x)$ and observe or even prove that its maximum is $y=25$ for $x=5$.

However, the fascinating thing is to note in the next step one way in which a mathematician, motivated by intellectual curiosity in this case, may move on to discover something new. Cardan says in line 3 (Fig. 2) *sic tamen operabimur*, "nevertheless we will operate," i.e., he says let's follow the procedures we used in other cases and see what happens. By completing the square he obtains $5+\sqrt{-15}$ and $5-\sqrt{-15}$. This is stated in line 8 using the abbreviations which characterized the "syncopated" algebra that was intermediary between the earliest period (sometimes called "rhetorical"), when all algebra was written out in words in full, and the modern "symbolic" period.

From Arabic times the proof of algebraic procedures had been given using geometric diagrams such as one sees in the "*Demonstratio*" in our Figure 2. In this case, however, Cardan had to resort in line 9 of this "*Demonstratio*" to *imaginaberis* ("You will imagine"). Although he gives below the diagram a computational check of his results by showing that $5+\sqrt{-15}$ times $5-\sqrt{-15}$ is $25-(-15)$, which is 40, in the discussion he notes that these quantities are *uere sophistica* ("truly sophisticated"), and in the last line of the "*Demonstratio*" he states that continuing to work with these numbers would involve one in an *Arithmetica subtilitas*—"as subtle as it would be useless."

Thus we see a man proceeding to obtain by analogy and formalism results which he finds interesting but which he does not fully comprehend nor accept himself. In more recent times Oliver Heaviside had a similar experience when he developed and used his operational calculus in spite of the criticism of some of his mathematical contemporaries.

The first statement of the rules for operating with square roots of negatives is shown in our Figure 3 taken from the 1579 edition of Rafael Bombelli's *L'Algebra parte maggiore del arithmetica* published in Bologna (1st edition, 1572). Cardan's formulas for the roots of cubic equations involve complex numbers, oddly enough, in the so-called irreducible case in which all three roots are real. He did not solve this case.

If $y^3+py+q=0$ Cardan's formula in modern form and symbols is $y=A+B$, where

$$A=\sqrt[3]{-\frac{q}{2}+\sqrt{\left(\frac{q}{2}\right)^2+\left(\frac{p}{3}\right)^3}},$$

$$B=\sqrt[3]{-\frac{q}{2}-\sqrt{\left(\frac{q}{2}\right)^2+\left(\frac{p}{3}\right)^3}}.$$

PRIMO. 169

Ho trouato un'altra ſorte di R.c.legate molto differenti dall'altre, laqual naſce dal Capitolo di cubo eguale à tanti,e numero, quando'l cubato del terzo delli tanti è maggiore del quadrato della meta del numero come in eſſo Capitolo ſi dimoſtrarà, laqual ſorte di R. q. ha nel ſuo Algoriſmo diuerſa operatione dall'altre, e diuerſo nome; perche quando il cubato del terzo delli tanti è maggiore del quadrato della metà del numero; lo eccesſo loro non ſi può chiamare ne più,ne meno, però lo chiamarò più di meno, quando egli ſi douerà aggiongere, e quando ſi douerà cauare, lo chiamerò men di meno, e queſta operatione è neceſſariſſima più che l'altre R.c. L.per riſpetto delli Capitoli di potenze di potẽze, accompagnati cõ li cubi,ò tanti,ò con tutti due inſieme, che molto più ſono li caſi dell'agguagliare doue ne naſce queſta ſorte di R. che quelli doue naſce l'altra, la quale parerà à molti più toſto ſofiſtica, che reale, e tale opinione hò tenuto anch'io, ſin'che hò trouato la ſua dimoſtratione in linee (come ſi dimoſtrarà nella dimoſtratione del detto Capitolo in ſuperficie piana) e prima trattarò del Moltiplicare, ponendo la regola del più & meno.

Più uia più di meno,fà più di meno.
Meno uia più di meno,fà meno di meno.
Più uia meno di meno, fà meno di meno.
Meno uia meno di meno,fà più di meno.
Più di meno uia più di meno,fà meno.
Più di meno uia men di meno,fà più.
Meno di meno uia più di meno,fà più.
Meno di meno uia men di meno fà meno.

Si

Figure 3

The other roots are $\omega A+\omega^2 B$ and $\omega^2 A+\omega B$ where $\omega=(-1+\sqrt{3}i)/2$. The "irreducible case" is that in which

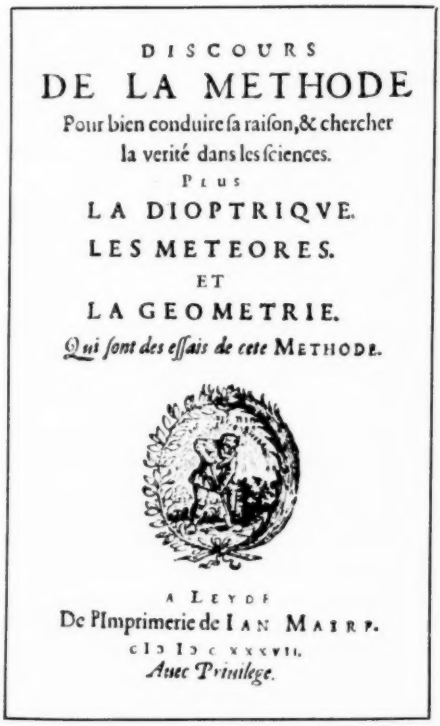
$$\Delta=\left(\frac{q}{2}\right)^2+\left(\frac{p}{3}\right)^3<0.$$

Bombelli showed how to combine the two complex numbers given by Cardan's formula in this latter case to obtain finally the correct real root. To do this he had to develop rules for operating with square roots of negatives. One can read his statement of these rules at the bottom of his page 169 by use of the following vocabulary:

piu: a positive (quantity)
uia: times
di meno: a square root of a negative number
meno: a negative (quantity)

Some perception of the difficulties involved in the "rhetorical" stage of algebraic symbolism may be obtained by looking at the end of the second and third lines of the page. Here Bombelli writes that he is treating "cubes equal to quantities and numbers" (i.e., $x^3=px+q$). Note that he uses a different word for each power of the unknown, and that he deliberately writes his equations so that there are no negative signs. He goes on to say, beginning in line 7, that he will treat the case in which *il cubato del terzo del li tanti è maggiore del quadrato della metà del numero* ("the cube of one third of the [coefficient of] x is greater than the square of one half the constant"). Since Bombelli's original equation is set up such that the p of our modern symbolic form of Cardan's formulas would be negative, this condition amounts to requiring that $\Delta<0$, the irreducible case noted above.

Figure 4 is the title page of René Descartes' most famous work. It emphasizes the connections between mathematics, philosophy, and logic, and the interesting fact that his *La Géométrie* was merely the third illustrative appendix to his *Discourse on the Method of Reasoning and Seeking Truth in Science*. (Note the old use of cIↃ and IↃ for *M* and *D* in writing Roman numerals.)


DISCOURS
DE LA METHODE
Pour bien conduire ſa raiſon,& chercher la verité dans les ſciences.
Plus
LA DIOPTRIQVE.
LES METEORES.
ET
LA GEOMETRIE.
Qui ſont des eſſais de cette METHODE.

A Leyde
De l'Imprimerie de Ian Maire.
cIↃ IↃ c xxxvii.
Auec Priuilege.

Figure 4

Figure 5 shows page 380 of this book and points out further that *La Géométrie* was far from being anything like our modern analytic geometry texts and that, in fact, it contained much that was purely algebraic. There are three further things to be remarked about this page. The middle paragraph and its marginal note contain the first use of "imaginary" as the name of these new numbers. They had earlier been described variously. (We have seen them called "sophisticated" and "subtle.") Unfortunately the term "imaginary" stuck and has come down to us as an historical hangover—and as a headache to teachers, since students naturally think of "imaginary" as a descriptive adjective rather than as merely an arbitrary name for a mathematical object

eſtoient $\frac{1}{3}$, 1, & $\frac{4}{3}$, & que celles de la premiere eſtoient $\frac{2}{3}\sqrt{3}$, $\frac{1}{3}\sqrt{3}$, & $\frac{4}{3}\sqrt{3}$.

Cõment on rend la quantité connuë de l'vn des termes d'vne Equation eſgale a telle autre qu'on veut.

Cete operation peut auſſy ſeruir pour rendre la quantité connuë de quelqu'un des termes de l'Equatiõ eſgale a quelque autre donnée, comme ſi ayant $x^3 * - bbx + c^3 \propto 0$. On veut auoir en ſa place vne autre Equation, en laquelle la quantité connuë, du terme qui occupe la troiſieſme place, a ſçauoir celle qui eſt icy bb, ſoit $3aa$, il faut ſuppoſer $y \propto x \sqrt{\frac{3aa}{bb}}$; puis eſcrire $y^3 * - 3aay + \frac{3a^3c^3}{b^3}\sqrt{3} \propto 0$.

Que les racines, tant vrayes que fauſſes peuuent eſtre reelles ou imaginaires.

Au reſte tant les vrayes racines que les fauſſes ne ſont pas touſiours reelles; mais quelquefois ſeulement imaginaires; c'eſt a dire qu'on peut bien touſiours en imaginer autant que iay dit en chaſque Equation; mais qu'il n'y a quelquefois aucune quantité, qui correſponde a celles qu'on imagine. comme encore qu'on en puiſſe imaginer trois en celle cy, $x^3 - 6xx + 13x - 10 \propto 0$, il n'y en a toutefois qu'vne reelle, qui eſt 2, & pour les deux autres, quoy qu'on les augmente, ou diminue, ou multiplie en la façon que ie viens d'expliquer, on ne ſçauroit les rendre autres qu'imaginaires.

La reduction des Equatiõs cubiques lorſque le probleſme eſt plan.

Or quand pour trouuer la conſtruction de quelque probleſme, on vient a vne Equation, en laquelle la quantité inconnuë a trois dimenſions; premierement ſi les quantités connuës, qui y ſont, contienent quelques nombres rompus, il les faut reduire a d'autres entiers, par la multiplication tantoſt expliquée; Et s'ils en contienent de ſours, il faut auſſy les reduire a d'autres rationaux, autant qu'il ſera poſſible, tant par cete meſme multiplication,

Figure 5

which is as real in its existence as a mathematical point or line. Note further that Descartes' *fausses racines* ("false roots") denoted negative, not imaginary, numbers. He apparently was not entirely willing to accept negatives as being as com-

pletely valid as positive numbers and did most of his geometrical drawing in what we would call the first quadrant. This illustrates how each extension of our number system from integers to fractions, to irrationals, to complex, to quaternions, to transcendentals has been first viewed with scepticism as an unreal or inapplicable abstraction. Acceptance and, eventually, fairly general understanding of such extensions have grown with familiarity, improved logical rigor and organization, graphical representation, and applications in both the mathematical and physical worlds. But these concepts, in their earliest days, bothered such geniuses as Descartes and Gauss.

All of these aspects appear in the growth of the acceptance of complex numbers. Their use in extending mathematics appears implicitly in the middle paragraph of our Figure 5 where Descartes is discussing the number of roots which an equation may have. Although, even before Descartes, Albert Girard and others had suggested that an *nth* degree equation may have n roots, it required a further expanded theory of complex numbers and their graphical representation before C. F. Gauss could prove the fundamental theorem of algebra, that every rational integral polynomial with real or complex coefficients has at least one root. From this the n root theorem follows directly.

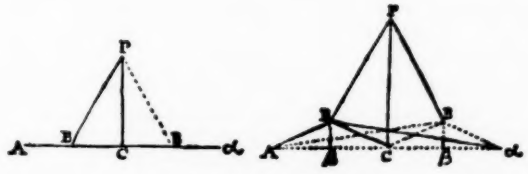
Figure 6 serves to illustrate several phases in the development of ideas, as well as to point up again the continuity and internationalism so common in the history of mathematics. This picture is from the 1693 Latin edition of John Wallis' *Algebra*, which first appeared in English in 1685. The lower diagrams speak for continuity and internationalism because they appeared first in the work of Descartes. They show a geometrical construction for the roots of a quadratic equation of the type $x^2 \mp bx - c = 0$. (However, Wallis did not use x as Descartes did, but wrote $aa \mp ba - æ = 0$, using vowels for the unknown as Viete had done earlier. Wallis regarded the arbitrary constants as always representing positive numbers.)

If $CP = \frac{1}{2}b$, and $PB = \sqrt{c}$ is perpendicular to CP, then the circle on CP determines the secant αB on which AB and $B\alpha$ are equal in length to the roots of the equation. (Note the connection between the geometric theorem that the product of the segments of a secant equals the square of the tangent from an external point, and the algebraic theorem that the product of the roots of this quadratic is c.)

The left-hand diagram at the top of the page gives Wallis' construction for the roots of $x^2 \mp bx + c = 0$ if $(b/2) \geqq \sqrt{c}$. Take $AC = \frac{1}{2}b = C\alpha$ and $PC = \sqrt{c}$ perpendicular to $A\alpha$ at C. The radius $PB = \frac{1}{2}b$ with center at P determines the points B. The roots are then the numerical values of the two lengths AB.

If $(b/2) < \sqrt{c}$, the above construction fails, in which case Wallis used PC and PB to construct a right triangle with the right angle at B, as shown in the upper right hand diagram, rather than at C. He then says that the points B which are *above the line* may be regarded as representing the solution in this case just as the *B's on the line* determined the solution when the roots were real. This, we see, was a step toward the modern graphical representation of complex numbers. Incidentally, recall that Descartes, whom Wallis had studied and admired, had interpreted imaginary roots for algebraic equations as indicating that the related geometric construction was impossible. Wallis also suggested representing $\sqrt{-1}$ by applying the Euclidean construction for a mean proportional to directed line segments representing $+1$ and -1, and he had argued that, just as people had become reconciled to representing both positive and negative numbers on a line and associating them with distances forward and backward, so one could imagine negative areas as land covered by rising water,[4] and use such a

[4] A reprint of these discussions may be found in *A Source Book in Mathematics* edited by D. E. Smith (New York: McGraw-Hill and Co., 1929) pp. 46 ff.



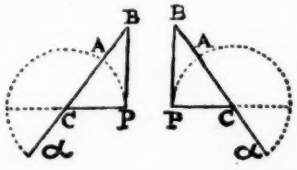
Utroque casu (sive ad C sive ad B sit rectus angulus) potest punctum B (ad utramvis partem rectæ PC) indifferenter sumi, simili situ. Duoque puncta B B sunt quæ designat hæc æquatio.

In primo casu; A B α est una linea recta, eademque cum A C α.

In posteriori casu; A B α tali ad B angulo incurvatur, ut punctorum A α distantia æquet rectam A C α; quæ est linea ichnographica, in quam si ichnographice projiciatur A B α, puncta B B in subjecta β β cadent.

Si igitur, in Problemate unde hæc emergit æquatio, supponatur A B α una recta, aut punctum B in ipsa A C α recta, aut cum β coincidere, aut A C α æqualis aggregato A B + B α, aut aliud quid quod horum aliquod includat: Indicat nec constructio, casum (sic intellectum) impossibilem esse; sed &, quomodo emperandus sit ut fiat possibilis.

Inter hanc impossibilitatem, eamque quæ lateralibus etiam æquationibus est communis, hæc est differentia. Quando in æquatione laterali, prodit radicis valor negativus; tantundem est ac dicere, Non posse quidem (verbi gratia) imperatum punctum B haberi (in expolita A C recta) prorsum (ut supponitur,) sed Retrorsum ab A haberi posse in exhibita distantia. Quando vero in æquatione quadratica prodit, non quidem valor Negativus, (quod huic cum laterali est commune,) sed (qui dici solet) valor imaginarius (ut est quadrati negativi, radix imaginaria;) tantundem est ac dicere; Non posse haberi imperatum punctum B in AC (utcunque producta) quod præsumitur; posse autem (extra eam lineam) in eodem plano haberi, infra *suprave* illam lineam.



Altera Æquationis Quadraticæ forma, $aa \mp ba - x^2 = o$, sic simplicissime constructur. Sumpta C A, vel C P, $= \frac{1}{2}b$; & P B $= \sqrt{x^2}$; angulum rectum ad P constituentibus: Hypotenusa B C connumerata, secabit circulum P A α in A α; eruntque desideratæ radices A B, B α: inter quas B P tangens est media proportionalis, & A α earum differentia sedulis signis. Sed radicum altera exponenda affirmative, altera negative. (Quoniam si A B intelligatur prorsum, erit B α retrorsum: si illa, retrorsum; hæc, prorsum.) Nimirum, + A B, — B α, si habeatur (in Æquatione) $+ba$; sed — A B, + B α, si habeatur $-ba$: adeoque A α utriusque lateris signis aggregatum.

Efficit hæc, hujus formæ, constructio maxime naturalis: propter B P (extra circulum) mediam proportionalem inter radices affirmativam & negativam: prout, in illa altera, P B (intra circulum) erat media proportionalis vel inter affirmativas ambas, vel ambas negativas.

Sed hæc constructio, ad hunc locum, haud spectat: Quoniam, in hac Æquationis forma, nunquam pervenitur ad hujusmodi valores marginales. Nam B P $= \sqrt{x}$, cujuscunque longitudinis, recta utcunque dati circuli Tangens est:

Figure 6

visualization to encourage the acceptance of negative squares.

Not long after Wallis, Lambert used complex variables in the development of map projections, and D'Alembert used them in the study of hydrodynamics. The stories of these and other applications, the story of the geometric representation of complex numbers, and the story of a modern rigorous formulation of an arithmetic for these numbers are interesting tales to which we will return later; but we hope that the story so far offers a way to an understanding of the numbers, their name, how and why they developed and the relative roles in their growth of application and abstraction, representation and generalization, practical necessity, and intellectual curiosity.

What's new

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BOOKS

SECONDARY

Fundamental Mathematics, Virgil S. Mallory and Kenneth C. Skeen, New York, Prentice-Hall, Inc., 1953. Paper, i+128 pp. $1.95.

Know Your Arithmetic,
Answers to Know Your Arithmetic with Additional Suggestions,
Know Your Geometry,
Answers to Know Your Geometry with Additional Problems and Suggestions,
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Answers to Know Your Algebra,
Know Your Trigonometry,
Answers to Know Your Trigonometry,
Wall-chart of Logarithms and Trigonometric Functions,
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• MATHEMATICAL MISCELLANEA

Edited by Paul C. Clifford, State Teachers College, Montclair, New Jersey, and Adrian Struyk, Clifton High School, Clifton, New Jersey.

Out of the mouths of babes . . .

by Paul C. Clifford

We sometimes are rewarded by having a student turn in a solution that is so neat we feel foolish for not thinking of it. In a trigonometry class we were discussing the area of triangles as a function of various given parts, and I suggested the following problem: A trough is constructed of two planks 12 inches wide. Find the width at the top for maximum volume. I then drew Figure 1 on the board, setting up the relation

$$A = x\sqrt{144 - x^2}.$$

I explained that an exact solution would require calculus, and found that x was approximately 8.5. At this point one of my students said that he could not see why calculus was needed, he had an exact solution. Being somewhat skeptical I asked him to put it on the board, which he did.

Figure 2 is his drawing. Since the area is $\frac{1}{2} \times 12 \times h$, the area will be a maximum when h is a maximum, which will be when h equals 12. The hypotenuse will then be $12\sqrt{2}$, which is the required dimension. Which proves that there is a great deal to the point of view, and that an old experienced teacher still has a lot to learn from the students.

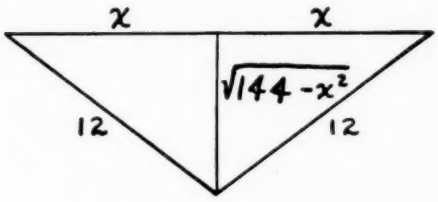

Figure 1

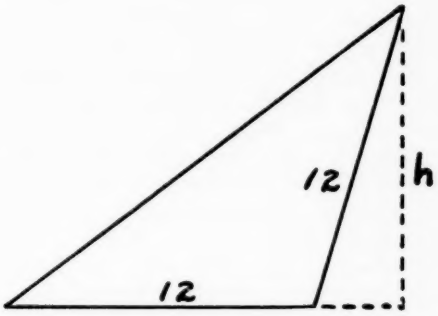

Figure 2

Elementary calculation of logarithms

Contributed by William R. Ransom, Tufts College, Medford, Massachusetts

A scheme to introduce logarithms so as to remove the mystery about possible origins of tables, and it emphasizes the fact that common logarithms are exponents of ten.

A student can calculate a table of logarithms in a very elementary way. To be sure they will be only approximate, but that is also true of the logarithms in our tables.

All that is needed is a knowledge of the meaning of fractional exponents and a little ingenuity in discovering suitable powers and products. We will use the symbol $\doteq$ for approximate equality.

Since $2^{10}=1024\fallingdotseq 10^3$, $10 \log 2 \fallingdotseq 3$, and $\log 2 \fallingdotseq .3$ with a relative error of $\frac{1}{3}$ of one per cent.

Since

$3^4=81\fallingdotseq 80, 4 \log 3 \fallingdotseq 1+3 \log 2, \log 3 \fallingdotseq .475.$

Since

$5 = 10 \div 2, \log 5 = 1 - \log 2 \fallingdotseq .7.$

Since

$7^2=49\fallingdotseq 50, 2 \log 7 = 1+\log 5, \log 7. \fallingdotseq 85.$

We need only compute primes in this way. Clues for the next ten primes follow. The decimals in parentheses denote the errors in fractions of one per cent.

$11^2=121\fallingdotseq 120$, $\log 11 \fallingdotseq 1.0375$, (.37)

$13\times7\times11=1001\fallingdotseq 10^3$, $\log 13 \fallingdotseq 1.11$, (.35)

$19\times4= 76\fallingdotseq 75$, $\log 19 \fallingdotseq 1.275$, (.29)

$23\times7=161\fallingdotseq 160$, $\log 23 \fallingdotseq 1.35$, (.87)

$29\times6=174\fallingdotseq 175$, $\log 29 \fallingdotseq 1.475$, (.87)

$31\times4=124\fallingdotseq 125$, $\log 31 \fallingdotseq 1.5$, (.58)

$37\times7=256\fallingdotseq 260$, $\log 37 \fallingdotseq 1.56$, (.52)

$41\times9=369\fallingdotseq 370$, $\log 41 \fallingdotseq 1.61$, (.17)

$43\times3=129\fallingdotseq 130$, $\log 43 \fallingdotseq 1.635$, (.09)

$47\times6=282\fallingdotseq 280$, $\log 47 \fallingdotseq 1.675$, (.17)

Quasi-right triangles

by Adrian Struyk,

The material which appears below is a digest of notes and problems cited in the bibliography at the end of the article. An attempt has been made to simplify, in some respects to amplify, and to present the subject in a form suitable for use in the high-school classroom or club.

Definition. In a quasi-right triangle the *difference* of two angles is equal to 90°.

Convention. In all that follows we shall take $\angle A = 90° + \angle B$.

Restriction. 45° is the upper limit for $\angle B$. For if $B = 45°$ then $A + B = 180°$.

Constructions. (1) Start with a right triangle ACE having the right angle at A. Extend hypotenuse CE through E to B, making $EB = EA$. Draw AB. Then $\angle B = \angle EAB$, and $\angle CAB = 90° + \angle B$.

(2) Start with a right triangle ABD having the right angle at A. Extend hypotenuse BD through D, and intersect this extension at C by a line so constructed that $\angle CAD = \angle B$. Then $\angle CAB = 90° + \angle B$.

Properties. (1) The bisector of $\angle C$ meets AB at a 45° angle.

Proof: From $A+B+C=180°$ subtract $A-B=90°$. Then $2B+C=90°$, from which $B+\frac{1}{2}C=45°$. Let the bisector of $\angle C$ meet AB at H. From triangle BCH exterior $\angle AHC = B+\frac{1}{2}C=45°$.

(2) The bisectors of the interior and exterior angles at C are equal.

Proof: These bisectors form a right angle since they bisect supplementary-adjacent angles. Let H and H' be the points at which the interior and exterior bisectors respectively meet AB and its extension. Triangle CHH' is an isosceles right triangle because the angles at C and H are 90° and 45° respectively. Hence the property.

(3) $c^2(a^2+b^2) = (a^2-b^2)^2$.

Proof: Let the line perpendicular to AB at A meet BC at D. As in the figure of construction #2 the angles CAD and B are equal. These angles and the common angle C make triangles DAC and ABC similar. Sides of triangle ABC are $BC=a$, $CA=b$, $AB=c$; and the corresponding sides of triangle DAC are $AC=b$, $CD=y$, $DA=x$. Hence $x:c=y:b=b:a$, making $x=bc/a$, $y=b^2/a$. In right triangle ABD $c^2+x^2 = (a-y)^2$. Substitute for x and y and simplify. The conclusion results.

(4) Conversely, if in triangle ABC $c^2(a^2+b^2) = (a^2-b^2)^2$, then $A-B=90°$.

Proof: From the given equation.

$$c\sqrt{a^2+b^2}=(a+b)(a-b).$$

Here the quantities other than $a-b$ are certainly positive. Hence $a-b$ is positive; that is, a exceeds b. With radius b, therefore, and with center C, draw a circle through A, cutting BC at K. Extend BA and BC to meet this circle at J and K' respectively. Draw JC. On secants BAJ and BKK', $BK=a-b$, $BK'=a+b$, $BA=c$, and let $BJ=z$. Now $BJ\cdot BA=BK'\cdot BK$. That is,

$$zc=(a+b)(a-b),$$

and

$$z^2c^2=(a^2-b^2)^2.$$

Comparing this with the equation of the hypothesis we get

$$z^2=a^2+b^2.$$

The triangle BCJ is therefore a right triangle, making $\angle J=90^\circ-\angle B$. But in the isosceles triangle ACJ $\angle J=\angle CAJ$ $=180^\circ-\angle BAC$. Consequently $90^\circ-B=180^\circ-A$, whence $A-B=90^\circ$.

(5) $2/c^2=1/(a+b)^2+1/(a-b)^2$.

Proof: From (3), $1/c^2=(a^2+b^2)/(a^2-b^2)^2$. Multiplying by 2, and rearranging,

$$\frac{2}{c^2}=\frac{a^2-2ab+b^2+a^2+2ab+b^2}{(a+b)^2(a-b)^2}.$$

Dividing as indicated yields the desired result.

(6) Parametric equations. In the figure described under property (3) delete letters x and y. Let $AB=r$, $AD=s$, $BD=t$, where $r^2+s^2=t^2$; then $CD=a-t$. From the similar triangles ABC and DAC,

$$a:b=r:s=b:(a-t);$$

and hence the independent equations

$$as=br,\quad bs=ar-rt$$

are obtained. Then, solving for a and for b, we may write $c=r$,

$$a=r^2t/(r^2-s^2),\ b=rst/(r^2-s^2).$$

(7) Integral values for a, b, c. If the parameters r, s, t are integers (i.e., Pythagorean numbers) the corresponding values of a and b are rational fractions, and c is an integer. To make a, b, c relatively prime integers multiply by the common denominator and divide by the common factor. Thus

$$a=rt,\quad b=st,\quad c=r^2-s^2.$$

(8) Since $A=90^\circ+B$ then also

$$2A=180^\circ+2B,\quad 3A=270^\circ+3B.$$

Hence

$$\sin A=\cos B,\qquad \cos A=-\sin B;$$

$$\sin 2A=-\sin 2B,\quad \cos 2A=-\cos 2B;$$

$$\sin 3A=-\cos 3B,\quad \cos 3A=\sin 3B.$$

(9) The functions of B are readily obtained from a figure drawn and labelled as under property (3). We find

$$\sin B=x/(a-y)=bc/(a^2-b^2);$$

$$\cos B=c/(a-y)=ac/(a^2-b^2);$$

$$\tan B=b/a.$$

We remark that the equation $\tan B=b/a$ also applies to a right triangle having legs a and b. See in connection with this the right triangle BCJ under property (4). The equation may be used as basis for the construction of a quasi-right triangle whose sides a and b are given.

(10) The functions of B may be obtained just as readily without reference to a figure. Since $\cos B=\sin A$ the law of sines $b/a=\sin B/\sin A$ gives $b/a=\tan B$. Then the identities

$$\sin B=\tan B/\sqrt{(1+\tan^2 B)}$$

$$\cos B=1/\sqrt{(1+\tan^2 B)}$$

lead to results as stated in (9). All values are positive because B is acute.

(11) For the functions of $2B$ the formulas are found to be

$$\sin 2B = 2abc^2/(a^2-b^2)^2 = 2ab/(a^2+b^2).$$

$$\cos 2B = c^2/(a^2-b^2) = (a^2-b^2)/(a^2+b^2).$$

(12) $c^2 = a^2 \cos 2B + b^2 \cos 2A$. To derive this interesting property clear the equation $c^2/(a^2-b^2) = \cos 2B$ of fractions and then substitute $\cos 2A$ for $-\cos 2B$.

(13) Functions of $3B$:

$$\sin 3B = c^3(3a^2b-b^3)/(a^2-b^2)^3;$$

$$\cos 3B = c^3(a^3-3ab^2)/(a^2-b^2)^3.$$

(14) $c^3 = a^3 \cos 3B + b^3 \cos 3A$. In (13) multiply the second result by a^3 and the first by b^3. Add these two products, and substitute $\cos 3A$ for $\sin 3B$. The coefficient of c^3 reduces to 1.

Exercises and problems.

#1. Find the angles of a quasi-right triangle which is also isosceles.

#2. Find the angles of a quasi-right triangle in which (a) $B = 2C$; (b) $C = 2B$; (c) $A = 2B$; (d) $A = 2C$; (e) the three angles are in arithmetic progression.

#3. Find the angles of an isosceles triangle such that the bisector of one of the equal angles divides the triangle into two quasi-right triangles.

#4. Show that if $A = nB$ then n exceeds 3 and $C = (n-3)B$.

#5. Show that if $B = nC$ then

$$A = (3n+1)C.$$

#6. One of the triangles into which a right triangle is divided by the bisector of an acute angle is a quasi-right triangle.

#7. Set up and prove the converse of the preceding statement.

#8. Prove that the converse of property (1) is true.

#9. Construct any or all of the triangles in problem #2.

#10. Construct a quasi-right triangle given . . . (any one side and any one angle may be given—nine possibilities).

#11. Construct a quasi-right triangle given sides a and b.

#12. In the figure of property (3) drop a perpendicular CP from C to line AB, and let $CP = h$. From similar right triangles

$$h : x = a : (a-y).$$

Hence show that

$$h = abc/(a^2-b^2).$$

#13. Let T denote the area of quasi-right triangle ABC. Show that

$$T = ab(a^2-b^2)/2(a^2+b^2).$$

#14. Let R denote the circumradius of quasi-right triangle ABC. Show that

$$R = (a^2-b^2)/2c.$$

(Recall that $abc = 4RT$ in any triangle.)

#15. Parametric expressions for R and T are easily derived. Show that

$$R = \tfrac{1}{2}t^2, \quad T = \tfrac{1}{2}rs(r^2-s^2).$$

#16. If the sides of quasi-right triangle ABC, taken in the order b, c, a, form an arithmetic progression, then

$$b/(\sqrt{7}-1) = c/\sqrt{7} = a/(\sqrt{7}+1).$$

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A column of unofficial comment.

An answer to Lee Emerson Boyer's paper

by Houston T. Karnes, Louisiana State University, Baton Rouge, Louisiana.

In his paper, "A New Responsibility of Teacher Education Programs,"[1] Professor Boyer has raised an issue which has been on the minds of many for several years. Professor Boyer recognizes the importance of the two-track program in the secondary-school curriculum and then proceeds to discuss the training of teachers to administer this program. His main thought is that there should be a two-track teacher-training program.

The best possible solution to problems of this nature is gained through the meeting of minds. Professor Boyer has produced one approach which shows that he has given much thought to the matter. This paper will set forth certain objections to the Boyer plan and will introduce another solution to the problem.

All of us are conscious of the problems which have been facing mathematics since 1920, though the inception of these problems appeared much earlier. We are happy over the progress that has been made. The two-track program seems to be a fine solution to some of our most difficult problems. However, unless it is administered correctly our work is for naught.

In a sense the two-track program tends to establish two departments where there was formerly one. It seems teachers will give instruction in either the specialized[2] or basic courses. Each of these needs different types of preparation. Unless we are careful the solution of a curriculum problem might be replaced by personnel and instructional problems. This could be more disastrous for our field than the troubles we are now experiencing. It has come to our attention on several occasions that teachers trained to teach the specialized courses resent being given classes in the basic courses. They seem to have a feeling that they are teaching materials beneath their station. This may not be widespread but the fact of its existence indicates a trend which is not good. Another thought that exists in the minds of some who are trained to teach specialized courses is that basic courses are more difficult to teach. This concept is no doubt true if one has not been trained to handle the subject matter of the basic courses. Professor Boyer implies as much in his paper. This being true, a reflection is cast upon those of us who train teachers for not providing the necessary preparation in this new area of instruction. One who knows mathematics, however, should be able to teach the basic subjects through extra work both in and outside of the class room. This fact does not release those who direct teacher training from the responsibility of providing adequate preparation and thus improving instruction.

With this introduction before us, a few objections to the Boyer plan will now be listed.

If there is danger of a split in the department at the instructional level a two-track training program would accentuate the condition. The extremely large number of small high schools could not afford nor justify employing mathematics teachers trained in only one phase of the two-

[1] See this issue p. 66.

[2] The words "specialized" and "basic" as used in this paper are defined in Professor Boyer's paper.

track program. A two-track teacher-training program would place an additional burden, which probably could not be justified, upon the collegiate departments which train teachers. In the event of a two-track teacher-training program there is a danger of the curriculum in the training of teachers for the basic courses being diverted away from good sound mathematical content.

All teachers of secondary mathematics should know mathematics well beyond the area wherein they are teaching. This means that teachers of both the specialized and basic courses should have training in pure mathematics. Those who teach the specialized courses should have the information Professor Boyer feels the teachers of the basic courses should have. This information can be used in the specialized courses to a great advantage in many ways.

It is therefore recommended that we have a one-track teacher-training program. This program should be developed to satisfy all necessary conditions as suggested. This can be done without lowering the standards of the present curriculum for the training of teachers of the specialized courses.

Many have thought for some time that the curriculum should be revamped. In fact, such changes have been under the consideration of various groups for the past few years.

The benefits of such a program would mean that all teachers would be qualified to teach either type of course. This would be advantageous both from an instructional and budgetary standpoint. It would mean, too, that all teachers would have a full appreciation of the relative importance of both specialized and basic courses.

Registrations at the Thirteenth Summer Meeting

Kalamazoo, Michigan, August 23–26, 1953

Arizona	1
Arkansas	1
California	1
Colorado	1
Connecticut	1
District of Columbia	1
Florida	2
Illinois	42
Indiana	44
Iowa	7
Kansas	2
Kentucky	4
Louisiana	2
Massachusetts	2
Michigan	147
Minnesota	4
Missouri	2
Nebraska	7
New Hampshire	1
New Jersey	2
New York	6
Ohio	41
Oklahoma	1
Oregon	2
Pennsylvania	8
South Dakota	1
Tennessee	1
Texas	1
Virginia	2
Washington	2
West Virginia	1
Wisconsin	29
Wyoming	1
Canada	8
Total	378

Fields of interest of the registrants as indicated by the check marks on the registration cards

High School	182
Junior High School	93
Teacher Training	70
Elementary	57
Supervision	30
College	29
Junior College	6
Number of registrants who are members of the Council	271

• REFERENCES FOR MATHEMATICS TEACHERS

Edited by William L. Schaaf, Department of Education, Brooklyn College, Brooklyn, New York

The editor presents a valuable guide to those teachers who are developing a library of mathematics for their school. His contribution is in response to a number of requests from readers of The Mathematics Teacher for information of this kind.

The high-school mathematics library.—I

The average high-school library usually carries on its shelves a goodly number of books on science. And why not? A science-conscious public is attracted by an abundance of popularized science literature, and the irresistible glamor of atomic energy, electronics, space ships, rockets, and the like. Regrettably, the same cannot be said for the mathematics shelf. To be sure, the celebrated man-in-the-street is not as conscious of mathematics, or, if so, he rather fears it; nor is the glamor as obvious. Yet there is no great dearth of books about mathematics for high school students and their teachers.

The following list, intended as a guide for librarians as well as for teachers and students, was compiled with these considerations in mind:

(1) standard textbooks have been excluded, except in the case of a few vocational subjects;

(2) professional books for teachers have been limited, in the main, to recent publications;

(3) out-of-print and otherwise inaccessible books have been held to a minimum.

To decide upon the appropriateness of a book was not always easy, and many a reader will doubtless disagree with this department on certain items. Our head is already rolling. . . .

Entries preceded by a bullet are items which probably should be in every high-school library. This is a matter of the compiler's judgment. No such collection ever met with everybody's approval.

We sincerely hope that this check list may serve to dispel any doubts concerning the bountiful number of books about mathematics, that it may serve to encourage young people to go to the library occasionally for a "math" book, that it may induce librarians to acquire additional books, and that it will serve as a useful guide to students and teachers.

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Author is a well-known writer on recreations of all sorts—fun with string, fun with paper, fun with magic, etc.

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MACMAHON, P. A. *New Mathematical Pastimes.* Cambridge: Cambridge Univ. Pr., 1930. Pp. 116.

MERRILL, HELEN A. *Mathematical Excursions.* Boston: Humphries, 1933. Pp. 145.

● MEYER, JEROME S. *Fun with Mathematics.* Cleveland & New York: World Pub., 1952. Pp. 176.
Sophisticated and attractive; contains considerable new material.

● MOTT-SMITH, GEOFFREY. *Mathematical Puzzles for Beginners and Enthusiasts.* Philadelphia: Blakiston, 1946. Pp. 248.
Unusually fine collection of mathematical recreations, well presented.

● NORTHROP, EUGENE. *Riddles in Mathematics.* New York: Van Nostrand, 1944. Pp. 262
Emphasis on mathematical paradoxes and fallacies, including paradoxes of logic, of probability, and of the infinite.

PHILLIPS, HUBERT. *Brush Up Your Wits.* Philadelphia: McKay, 1936. Pp. 116.

PHILLIPS, HUBERT. *Question Time: An Omnibus of Problems for a Brainy Day.* New York: Farrar & Rinehart, 1938. Pp. 265.

PHILLIPS, H., SHOVELTON, S., and G. MARSHALL. *Caliban's Problem Book.* London: T. De la Rue & Co., 1933. Pp. 330.

PROSKAUER, J. *Puzzles for Everyone.* New York: Harper, 1944. Pp. 176.

ROW, T. SUNDARA. *Geometric Exercises in Paper Folding.* Chicago: Open Court, 1941. Pp. 148.

RULON, P. J. *Brain Teasers; or, Hurdles for the Mental Athlete.* Boston: Page, 1932. Pp. 250.

● STEINHAUS, HUGO. *Mathematical Snapshots.* New York: Oxford Univ. Pr., 1950. Pp. 266.
A unique collection of interesting mathematical facts; expository and recreational material.

● STORME, P. and STRYFE, P. *How to Torture Your Friends.* New York: Simon & Schuster, 1941. Pp. 170.
Delightful assortment of brain twisters, puzzles, fallacies, tricks, quizzes, and quips, attractively presented.

WILLIAMS, W. T. and G. H. SAVAGE. *The Penguin Problems Book.* New York: Penguin Books, 1940. Pp. 156.
Collection of 100 provocative inferential and mathematical problems, and 100 word problems (acrostics, anagrams, word squares, etc.).

WILLIAMS, W. T. and G. H. SAVAGE. *The Second Penguin Problems Book.* New York: Penguin Books, 1944. Pp. 467.

● YATES, R. C. *The Trisection Problem.* Ann Arbor: Edwards Bros., 1942, 1947. Pp. 68.
A refreshing treatment of a hoary problem; many solutions, historical notes, interesting sidelights.

4. EXPOSITORY MATHEMATICS; POPULAR ACCOUNTS

● BAKST, AARON. *Mathematics: Its Magic and Mastery.* New York: Van Nostrand, 1952. Pp. 790.
A comprehensive and readable exposition.

● BELL, E. T. *The Handmaiden of the Sciences.* Baltimore: Williams & Wilkins, 1937. Pp. 216.
Brilliant exposition of the role of mathematics in the physical sciences.

BELL, E. T. *The Magic of Numbers.* New York: McGraw-Hill, 1946. Pp. 418.

● BELL, E. T. *Mathematics: Queen and Servant of Science.* New York: McGraw-Hill, 1951. Pp. 437.
A revision and amplification of *The Queen of the Sciences* and *The Handmaiden of the*

Sciences: skillfully rewritten into one volume.

● Bell, E. T. *The Queen of the Sciences.* Baltimore: Williams & Wilkins, 1931. Pp. 138.
A charming classic; deftly written, and highly illuminating.

● Dantzig, Tobias. *Number: The Language of Science.* New York: Macmillan, 1949. Pp. 320.
Deservedly popular exposition of the concept of number and its development; difficult reading in spots.

Dubisch, Roy. *The Nature of Number.* New York: Ronald Pr., 1952. Pp. 159.
Concise and lucid; for advanced readers.

● Hardy, G. H. *A Mathematician's Apology.* Cambridge: Cambridge Univ. Pr., 1940. Pp. 93.
An inimitable essay, sincere and significant.

● Hooper, Alfred. *The River Mathematics.* New York: Holt, 1945. Pp. 401.

● Kasner, E. and J. Newman. *Mathematics and the Imagination.* New York: Simon & Schuster, 1940. Pp. 380.
One of the best popular expositions of mathematics ever written.

Katsoff, L. O. *A Philosophy of Mathematics.* Ames: Iowa State College Pr., 1948. Pp. 266.

Keyser, C. J. *The Human Worth of Rigorous Thinking.* New York: Scripta Mathematica, 1940. Pp. 323.

● Kline, Morris. *Mathematics in Western Culture.* New York: Oxford Univ. Pr., 1953. Pp. 484.

● Kramer, Edna E. *The Main Stream of Mathematics.* New York: Oxford Univ. Pr., 1951. Pp. 319.
A popular account of the nature, development, and significance of mathematics.

● Lieber, L. R. and H. G. Lieber. *The Education of T. C. Mits.* New York: Norton, 1944. Pp. 230.
Unique style and unusual drawings; whimsical yet sober exposition of the fundamental nature of mathematics.

Lieber, L. R. and H. G. Lieber. *Mits, Wits and Logic.* New York: Norton, 1947. Pp. 240.
Another sparkling and sophisticated exposition.

● Lieber, L. R. and H. G. Lieber. *Take a Number: Mathematics for the Two Billion.* Lancaster, Pa.: Jaques Cattell Pr., 1946. Pp. 221.
A novel introductory survey of elementary algebra and geometry.

● Read, A. H. *A Signpost to Mathematics.* (Thrift Books, No. 8). London: C. A. Watts & Co., Ltd., 1951. Pp. 123.
Excellent discussion of principles which guide the mathematician in his thinking.

● Schaaf, W. L. (ed.). *Mathematics—Our Great Heritage. Essays on the Nature and Cultural Significance of Mathematics.* New York: Harper, 1948. Pp. 291.
Foundations, philosophy, humanism; for the more mature reader.

Van Horn, C. E. *A Preface to Mathematics.* Boston: Chapman & Grimes, 1938. Pp. 124.
Addressed to teachers, but contains much of interest to the general reader.

5. Survey Mathematics

● Anderson, R. W. *Romping Through Mathematics.* New York: Knopf, 1947. Pp. 152.

● Boyer, Carl B. *The Concepts of the Calculus.* New York: Columbia Univ. Pr., 1939; Hafner Pub. Co., 1949. Pp. 346.

● Boyer, Lee Emerson. *An Introduction to Mathematics for Teachers.* New York: Holt, 1945. Pp. 478.
Fundamentals of arithmetic, algebra, geometry and trigonometry; historical backgrounds and practical applications.

● Courant, R. and H. E. Robbins. *What Is Mathematics: an Elementary Approach to Ideas and Methods.* New York: Oxford Univ. Pr., 1941. Pp. 521.
For mature readers; excellent discussion of the number system, geometric constructions, postulational systems, number theory, topology.

De Bray, M. E. J. G. *Exponentials Made Easy.* New York: Macmillan, 1921, 1928.

● Fehr, Howard. *Secondary Mathematics: A Functional Approach for Teachers.* Boston: Heath, 1951. Pp. 431.

● Hogben, Lancelot. *Mathematics for the Million.* New York: Norton, 1937. Pp. 647.
A pioneer best-seller among popular books on mathematics; more demanding upon the reader than its title suggests.

Jones, Burton W. *Elementary Concepts of Mathematics.* New York: Macmillan, 1947. Pp. 294.
For college students who have had little or no previous training in mathematics.

● Kokomoor, F. W. *Mathematics in Human Affairs.* New York: Prentice-Hall, 1942. Pp. 754.
An outstanding piece of popular exposition.

● Logsdon, M. I. *A Mathematician Explains.* Chicago: Univ. of Chicago Pr., 1936. Pp. 189.

Miller, Denning. *Popular Mathematics.* New York: Wise, 1942. Pp. 616.

Moore, J. H. and J. A. Mira. *The Gist of Mathematics.* New York: Prentice-Hall, 1942. Pp. 726.

● Richardson, M. *Fundamentals of Mathematics.* New York: Macmillan, 1941. Pp. 525.

Sawyer, W. W. *Mathematicians' Delight.* New York: Penguin Books, Inc., 1946. Pp. 215.

Sawyer, W. W. (and others). *Mathematics in Theory and Practice.* (New Education Library.) New York: Wm. H. Wise & Co., 1952. Pp. 384.

● Shackle, G. L. S. *Mathematics at the Fireside.*

Cambridge: Cambridge Univ. Pr., 1952. Pp. 156.
A delightfully fresh presentation of some basic ideas of elementary mathematics.

THOMPSON, S. P. *Calculus Made Easy.* New York: Macmillan, 1927. Pp. 301.

● WHITEHEAD, A. N. *An Introduction to Mathematics.* New York: Holt, 1911; Oxford Univ. Pr., 1948. Pp. 256.
A well-known classic; sets forth basic concepts in simple, unaffected language.

6. ARITHMETIC

● ANDREWS, F. EMERSON. *New Numbers: How Acceptance of a Duodecimal Base (12) Would Simplify Mathematics.* New York: Harcourt Brace, 1935, 1944. Pp. 168.

● BAKST, AARON. *Arithmetic for Adults.* New York: Crofts, 1944. Pp. 319.

● BENDICK, JEANNE. *How Much and How Many?* New York: Whittlesey House, 1947. Pp. 188.
An unusual and attractive account of the development of weights and measures; cleverly illustrated.

● BUCKINGHAM, B. R. *Elementary Arithmetic: Its Meaning and Practice.* Boston: Ginn, 1947. Pp. 744.

BUTLER, CHARLES H. *Arithmetic for High Schools.* Boston: Heath, 1953. Pp. 336.

HOOPER, ALFRED. *An Arithmetic Refresher.* New York: Holt, 1944. Pp. 119.

● LARSEN, HAROLD. *Arithmetic for Colleges.* New York: Macmillan, 1950. Pp. 275.

● MCKAY, HERBERT. *Odd Numbers, or Arithmetic Revisited.* Cambridge: Cambridge Univ. Pr., 1940. Pp. 215.

MCKAY, HERBERT. *The World of Numbers.* Cambridge: Cambridge Univ. Pr., 1946. Pp. 198.

● NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS. *The Metric System of Weights and Measures.* Twentieth Yearbook, 1948. Pp. 303.
Survey of contemporary metric usage; historical backgrounds.

7. GEOMETRY

BARAVALLE, H. v. *Geometry at the Junior High School Grades and the Waldorf School Plan.* Garden City, N. Y.: The Waldorf School, Adelphi College, 1948. Pp. 32.

BRESLICH, ERNST R. *Diagrams in Three Dimensions for Solid Geometry.* New York: Newson & Co., 1939. 15 plates.
Use of anaglyphs with orthoscope.

BRESLICH, ERNST R. *Excursion in Mathematics.* Chicago: Orthovis Pub. Co., 1938. Pp. 47.
Solid geometric figures as seen through an orthoscope; anticipated "3-D" comic books by a decade.

● FRAME, J. S. *Solid Geometry.* New York: McGraw-Hill, 1948. Pp. 339.
Not a conventional textbook; original approach to 3-dimensional space relations; maps and projections.

● KERN, W. F. and J. R. BLAND. *Solid Mensuration with Proofs.* New York: Wiley, 1938. Pp. 172.
Very practical and rather complete.

● LINES, L. *Solid Geometry.* London: Macmillan, 1935. Pp. 292.
Material on polyhedrons, semi-regular and star polyhedrons, and crystal forms.

LOOMIS, ELISHA. *The Pythagorean Theorem.* Berea, Ohio: Mohler Printing Co., 1927. Pp. 214.
Collection of over 167 algebraic and geometric proofs of this famous proposition.

MAXWELL, E. A. *Geometry for Advanced Pupils.* New York: Oxford Univ. Pr., 1949. Pp. 176.

● MENGER, KARL. *You Will Like Geometry.* (A Guide Book for the Illinois Institute of Technology Geometry Exhibition at the Museum of Science and Industry.) Chicago, 1952. Pp. 34.
A delightful and stimulating pamphlet—all too brief.

MUELLER, CLARA H. *Geometric Concepts.* New York: Wiley, 1931. Pp. 205.

SHIVELY, L. S. *An Introduction to Modern Geometry.* New York: Wiley, 1939. Pp. 167.
The abler students will need only a background of elementary geometry.

● SPENCER, WALTER. *Basic Principles of Analytic Geometry.* Chicago: Orthovis Pub. Co., 1939. Pp. 27.
Effective use of anaglyphs.

● TAYLOR, E. H. and G. C. BARTOO. *An Introduction to College Geometry.* New York: Macmillan, 1949. Pp. 143.
A brief, attractive sequel to elementary geometry.

WOLFE, H. E. *Introduction to Non-Euclidean Geometry.* New York: Dryden Pr., 1945. Pp. 247.

● YATES, R. C. *Geometric Tools: A Mathematical Sketch and Model Book.* St. Louis: Educational Pubs., 1949. Pp. 194.
Suggestive and stimulating.

(To be concluded in the March issue)

Reviews and evaluations

Edited by Cecil B. Read, University of Wichita, Wichita, Kansas.

Calculus, C. R. Wylie, Jr., New York, McGraw-Hill Book Co., Inc., 1953. x+565 pp. $6.00.

This text contains the usual subject matter of a first course in calculus including an introduction to differential equations. Taylor's series in several variables, Fourier's series, and Abel's identity are among topics which are not usually found in an elementary textbook.

An introductory chapter "The Calculus in Perspective" should be of help in orienting the student to the subject and should thus avoid the shock which many students have when they are confronted at the first class session with a rather abstract discussion of limits and continuity.

While integration is not introduced in the text as early as in several recent books, it is so placed that some of it could surely be considered during the first quarter or semester. Some of the applications of differentiation, such as work on motion, are deferred until after integration is introduced making possible a more unified treatment.

The typography and drawing are good, the paging is well done, and the book appears to be very inviting to use as a text.—*Cleon C. Richtmeyer, Central Michigan College of Education, Mt. Pleasant, Michigan.*

Chemical Calculations—An Introduction to the Use of Mathematics in Chemistry, Sidney W. Benson, New York, John Wiley & Sons, Inc., 1952. v+217 pp. Cloth, $2.95.

For teachers of high school or college general mathematics and algebra this is an excellent source book of sensible uses of measurement, significant figures, equations, proportions and ratio, exponents, and scientific notation. It is organized around chemical units rather than mathematical units.—*Lyman C. Peck, Iowa State Teachers College, Cedar Falls, Iowa.*

Complex Analysis, Lars V. Ahlfors, New York, McGraw-Hill Book Co., Inc., 1953. xi+247 pp. $5.00.

Here is a book which is surely destined to become a standard text for a first-year graduate course in the theory of functions of a complex variable. The subject is treated with clarity and elegance and well illustrated with numerous problems. The emphasis throughout is on the geometric approach and power series are not introduced until the middle of the book. In addition to a thorough treatment of the standard material of such a course, some topics not usually found in books of this type are discussed, in particular a chapter on the Dirichlet Problem including a discussion of Perron's method.—*W. M. Boothby, Northwestern University, Evanston, Illinois.*

Differential Equations, Robert C. Yates, New York, McGraw-Hill Book Co., Inc., 1952. v+215 pp. Cloth, $3.75.

Any new text in differential equations needs justification in the face of so many books on the subject. Complete, yet brief, this text stands out in the field because of its consideration of the student in his approach to the subject. He is led to see the problem in its physical or geometrical context. Then it is analyzed mathematically. Finally a solution is developed which places the techniques of differential equations in their proper place.—*Lyman C. Peck, Iowa State Teachers College, Cedar Falls, Iowa.*

Geometry and the Imagination, David Hilbert and S. Cohn-Vossen, New York, Chelsea Publishing Co., 1952. ix+357 pp. $5.00.

This is a translation of the famous *Anschauliche Geometrie* of Hilbert and Cohn-Vossen, and it represents a most welcome addition to the semi-popular books on mathematics in English. The purpose of the book is to present, in as intuitive and pictorially clear a fashion as possible, without cumbersome analytical accompaniment, several well-chosen, illustrative topics and problems from diverse fields of geometry. Included in the exposition are conic sections, lattices and crystals, projective configurations, differential geometry, kinematics and topology. The treatment of these selected topics is most original in many cases, and is so presented that a person with a minimum of formal training can follow the reasoning. Even the professional mathematician will find much that is new in familiar subjects (eleven characterizations of the sphere, to pick an example at random). This book will be an important addition to the libraries of both interested laymen and practicing or teaching professional mathematicians.—*W. M. Boothby, Northwestern University, Evanston, Illinois.*

Introduction to Concepts and Theories on Physical Science. Gerald Holton. Cambridge, Massachusetts, Addison-Wesley Press, Inc., 1952. xviii+650 pp. $6.50.

This book is the result of the author's experience in teaching introductory physics to students not primarily planning to become professional physicists. The book deals in a novel

manner with the subject matter of physics, developing the various concepts from their historical and philosophical significance. Part A deals with the study of motion, stressing Galileo's analysis of falling and projected bodies. The initial treatment of velocity and acceleration will give teachers of mathematics an illuminating insight into how and why geometry and algebra become the tools and communicating agent for teaching physical concepts. Part B takes up the study of forces beginning with Newton's laws, introducing vector algebra (a much needed addition to our present high-school program in mathematics) and a study of harmonic motion and wave propagation. Part C treats planetary motion and gravitation. Part D is especially illuminating of method in the physical sciences (not teaching methods). The remainder of the book treats the theory of conservation, atomic theory in physics and chemistry, quantitative theory, and the nuclear atom.

One cannot study this book without feeling genuine admiration for the lucidity of development of physical concepts. Problems are interspersed throughout the text to give practice in thinking about concepts. Mathematics teachers will find in this book myriads of examples of their subject in application and a consolation that a genuine understanding of physics requires a genuine understanding of mathematics also.—*Howard F. Fehr, Columbia University Teachers College.*

Mathematics for Success, Mary A. Potter, Anne L. Neitzel, Dorothy A. Root, and Frances C. Enright. Boston, Ginn & Company, 1952. viii+440 pp., $2.68.

This book is the more advanced second course designed to follow *Mathematics to Use* by Potter, Dunn, Allen, and Goldthwaite. *Mathematics for Success* cuts across the fields of arithmetic, algebra, geometry, and trigonometry. The Check List of the Commission on Post-War Plans of the National Council of Teachers of Mathematics served as a guide in the selection of topics.

The outstanding features are: (1) an emphasis upon the estimating of answers before solving problems; (2) a chapter on the micrometer, which will be of particular interest to boys; (3) a guide to the pronunciation of terms as each term is introduced into discussions; (4) an abundance of practice exercises; (5) a large number of attractive and appropriate illustrations and diagrams; (6) a summary test at the end of each chapter; (7) a picture dictionary of words and terms.

Mathematics limited to the level of this book would be strictly a mechanical process that is done "according to the rule." For this reason, the book should be used by only those teachers who, from their own background, know how to develop basic understandings of fundamental operations and how to teach relationships.—*Julia Adkins, Columbus, Ohio.*

Mathematics of Finance, Edward R. Mouzon, Jr., and Paul K. Rees. Boston, Ginn & Co., 1952. viii+255 pp. +147 pp. of tables, $4.60.

The topics usually covered in a course in mathematics of finance are found in this text, including simple interest, compound interest, annuities, amortization, sinking funds, depreciation, bonds, life annuities and insurance. There are also appendices covering the topics of exponents, logarithms, progressions, and the binomial theorem. The tables include the usual interest and annuity tables and a five-place table of logarithms. The reviewer noted the absence of a six-place table of logarithms of interest-ratios. This omission may be justified by the emphasis on interpolation methods in finding unknown rates and times.

Features of note include (1) a mortality table using an interest rate of 2½%; (2) the careful attention given to terminology and definitions at the beginning of each chapter; (3) a summary of formulas and a set of review exercises at the end of each chapter. The text is pleasing in appearance, the typography is good, and answers to all exercises have been included. —*Cleon C. Richtmeyer, Central Michigan College, Mt. Pleasant, Michigan.*

Methods of Statistical Analysis (Second ed.), Cyril H. Goulden. New York, John Wiley & Sons, Inc., 1952. ii+457 pp., $7.50.

This is a revised edition of the original text published in 1939. This book is aimed primarily at the man who will eventually go into statistical research. The fundamental topics are handled in the usual manner, but the major emphasis of the text is on experimental design and some rather advanced methods of analysis; for example, incomplete block experiments and probit analysis.

To be able to fully utilize this text, the reader must either have a thorough knowledge of statistical fundamentals, or else spend sufficient time mastering the first seven chapters of this book. If this book were to be used as a class text, at least two semesters would be required to sufficiently cover it. As a reference book in statistics it is excellent. The last chapter deals with quality control and offers a short and rather complete introduction to the subject. This book is a worthy addition to the Wiley Statistics Series.—*William E. Felling, Parks College of Aeronautical Technology, East Saint Louis, Illinois.*

A Refresher Course in Mathematics, F. J. Camm, New York, Emerson Books, Inc., 1953. Cloth, v+240 pp. $2.95.

This is an English text of the "handbook" variety. It includes material from arithmetic, algebra, trigonometry, and calculus. Examples are given for each of the mathematical topics presented—many of the applications being taken from the science field. The book is in-

tended as a quick review of the basic undergraduate mathematics particularly applicable to engineering for those who at one time had mastered the material. Formulas for the principles of mechanics, definitions of mathematical and physical terms and units, tables of mensuration, and short trigonometric and logarithmic tables are included.

The usefulness of the text is limited to those who want a compact reference for basic undergraduate mathematics.—*Lester R. Van Deventer, Eastern Illinois State College, Charleston, Illinois.*

Solid Geometry, William G. Shute, William W. Shirk, and George F. Porter. New York, American Book Company, 1954. Cloth, vii +280 pp., $2.48.

This text provides the student with enough basic material and ample original problems for a full and interesting course in solid geometry. The proofs of the important theorems and the more difficult corollaries are given in full, written in logical, simple, and concise language. Because of this fact and the helpful suggestions for the solution of the originals and the illustrative problems, this book should be a very teachable one, enabling the student to proceed "on his own" with a minimum of help from the instructor.

The drawings illustrating the definitions, theorems, and problems are simple and clearly represent the three-dimensional figures. Brief historical notes and practical problems are distributed throughout the book. Review exercises, practice tests, and well-constructed examinations are included. The appendix contains additional theorems, a brief introduction to trigonometry, a short review of logarithms, and an outline of references to plane geometry.

The application of Cavalieri's theorem in deriving the formula for the volume of a prism, the presentation of the subject of locus, the theory of limits, as well as many other factors make this an outstanding textbook.—*Kathleen O'Donnell, University of Kansas, Lawrence, Kansas.*

Squaring the Circle and Other Monographs (Squaring the Circle, E. W. Hobson; Ruler and Compass, H. P. Hudson; The Theory and Construction of Non-Differentiable Functions, A. N. Singh; How to Draw a Straight Line: A Lecture on Linkages, A. B. Kempe), New York, Chelsea Publishing Co., 1953. Cloth, ii +57 pp.; ii +143 pp.; vii +110 pp.; i +51 pp., $3.25.

The four monographs reprinted in one volume each originally appeared as a single publication, but recently have not been readily available. The selection seems a little odd, for the same groups will not be interested in the four monographs. Portions of the first two and the fourth will be of interest to high-school teachers and students; this will not be true of the third. Historical material on the circle squaring problem is exceptionally good; it is unfortunate that some notes on recent developments, such as the computation of π by modern calculating machines, could not have been included.

Some interesting material for mathematics club programs is included. As a single example, the ruler and compass solution of the problem of the shortest path by which a spider in one corner of a room may reach a fly in the opposite corner of the ceiling.—*Cecil B. Read.*

Substance and Function and Einstein's Theory of Relativity, Ernst Cassirer (trans. from the German by William Curtis Swabey and Marie Collins Swabey), New York, Dover Publications, Inc., 1953. xii +465 pp. $1.95.

This is an excellent translation of a scholarly analysis of how people construct concepts. The book treats philosophical problems of knowledge and reality arising in the mathematical and physical sciences. It is difficult material, but teachers of mathematics will find that it is well worth the pains required to read it.—*Irvin H. Brune, Iowa State Teachers College, Cedar Falls, Iowa.*

A Survey of Modern Algebra (Rev. ed.), Garrett Birkhoff and Saunders MacLane. New York, The Macmillan Company, 1953. v +469 pp. $6.50.

The present edition represents a refinement of an already highly useful text. The original comprehensive *Survey* has been reordered somewhat and augmented to the extent of approximately fifty pages. Only the last five chapters remain unchanged.

Instructors who have used the original edition with college classes appreciate its scope. Those desiring a text replete with possibilities for courses tailored to various kinds of students should welcome this new edition.

Teachers of mathematics in secondary schools may want this book in their personal libraries. Probably the best way to appreciate the vitality and growth of mathematics today is to study modern algebra. Nowhere can teachers better catch today's spirit of mathematics. Moreover, many of the examples in this text might help teachers to communicate this spirit to their students.—*Irvin H. Brune, Iowa State Teachers College, Cedar Falls, Iowa.*

Trigonometry, John F. Randolph. New York, The Macmillan Company, 1953. v +220 pp. $3.00.

This is a condensed trigonometry text utilizing the general angle approach. Somewhat novel appendixes containing reviews of elementary algebra and directly related geometry and supplementary work on logarithms are included. Although the book is complete in scope and scholarly written, the condensing of textual material may have made the text difficult for the average and below-average student. It is definitely a college level text and appears too difficult for high school use.—*Arthur J. Hall, San Francisco State College, San Francisco, California.*

• WHAT IS GOING ON IN YOUR SCHOOL?

Edited by John A. Brown, University of Wisconsin, Madison 6, Wisconsin, and Houston T. Karnes, Louisiana State University, Baton Rouge 3, Louisiana.

The editors offer a report on the Los Angeles City College third annual William B. Orange Mathematics Competition for high school students. The report was made by Ben Gold, Competition Committee Chairman. Also, Mary White Duke reports on her method of teaching algebra. This is a report of a "taking stock" process which all teachers need to do occasionally.

Los Angeles City College Mathematics Prize Competition

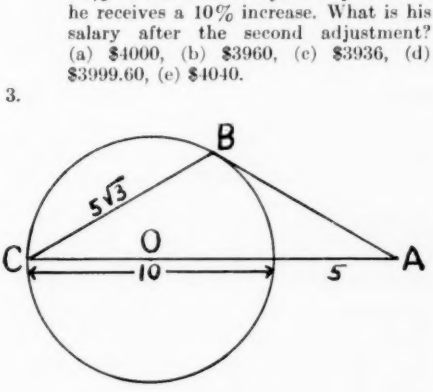
Contributed by Ben Gold, Competition Committee Chairman, Mathematics Department, Los Angeles City College, Los Angeles, California.

During May, 1953, Los Angeles City College held its third annual William B. Orange Mathematics Prize Competition for high school students of Los Angeles City High Schools.[1] Out of thirty-one schools entering, thirty had full teams. The three students with the highest score from a group of five were considered the team representing the school. This year North Hollywood High School won the team trophy. There were individual prizes consisting of cash, slide rules, books, and subscriptions to professional magazines awarded to various students.

Questions used for the competition were:

PART I (50%)—ONE HOUR

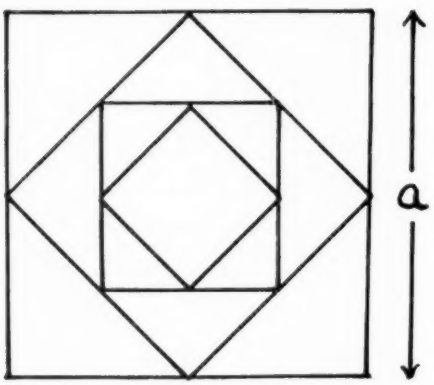
(Best answer to be checked on answer sheet.)

1. Machine A can make 5 articles in 10 minutes and machine B can make 10 articles in 5 minutes. How long will it take to make 1000 articles if both work together? (a) 800 minutes, (b) 400 minutes, (c) 200 minutes, (d) 100 minutes, (e) none of these answers.
2. A man who earns \$4000 per year receives a 10% decrease in salary. One year later he receives a 10% increase. What is his salary after the second adjustment? (a) \$4000, (b) \$3960, (c) \$3936, (d) \$3999.60, (e) \$4040.
3. 

AB is tangent to circle O at B. The length of AB in the figure is: (a) 5, (b) $5\sqrt{3}$, (c) $5\sqrt{6}$, (d) $10\sqrt{3}$, (e) 10.
4. The center of the inscribed circle of a triangle is the point of intersection of the: (a) medians, (b) angle bisectors, (c) altitudes, (d) perpendicular bisectors of the sides, (e) none of these answers.

[1] The first annual competition was reported in "What's Going on in Your School?" THE MATHEMATICS TEACHER, XLV, No. 1 (January, 1952), 34 and 35. The second annual competition appeared in the same department of THE MATHEMATICS TEACHER, XLV No. 7 (November, 1952). 536 and 542.

5.



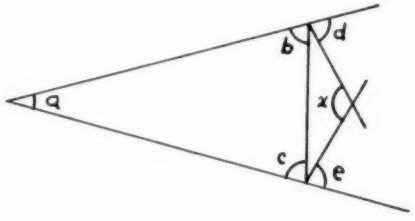
The figure consists of squares. The total length of all line segments in the figure is: (a) $12a$, (b) $8a$, (c) $6a\sqrt{2}$, (d) $6A(1+\sqrt{2})$, (e) $3a(2+\sqrt{2})$.

6. A rectangular box has dimensions of 3, 4, and 12. What is the length of the longest diagonal? (a) $\sqrt{19}$, (b) 12, (c) 13, (d) 18, (e) 19.
7. Which of the following is the largest? (a) $27^{-2/3}$, (b) 10^{-1}, (c) $4^{-3/2}$, (d) $10/99$, (e) $(3/2)^\circ \div 10$.
8. What is the number which, if decreased by 20%, will give a number 25% less than the number of which 100 is 90%. (a) 95, (b) 100, (c) 105, (d) 90, (e) 104 1/6.
9. By interchanging 2 adjacent digits, the number 14149686 can be made into a number divisible by 66. The two consecutive numbers are: (a) 41, (b) 14, (c) 49, (d) 96, (e) 68.
10. $\sqrt{19-4\sqrt{21}}$ is equivalent to: (a) $19-2\sqrt{21}$, (b) $\sqrt{12}-\sqrt{7}$, (c) $\sqrt{12}+\sqrt{7}$, (d) $\sqrt{19}+2\sqrt{21}$, (e) none of these answers.
11. If $m=a^x$, $n=a^y$, $a=(m^y n^x)^{z/2}$, then: (a) $m/n=1$, (b) $mn=1$, (c) $xy/z=1$, (d) $xyz=1$, (e) $mn=xy$.
12. Given that $\log 2=0.30103$, then $\log 25$ is: (a) 1.30103, (b) 1.37629, (c) 1.39794, (d) 0.39794, (e) none of these answers.
13. The value of $(1+i)^8$, where $i=\sqrt{-1}$, is: (a) $1+i^8$, (b) 2, (c) -1, (d) 16, (e) 64.
14. The quadratic equation which has the roots $1+\sqrt{3}$ and $1-\sqrt{3}$ is: (a) $x^2-3=0$, (b) $x^2-x-3=0$, (c) $x^2+x-3=0$, (d) $x^2-2x-2=0$, (e) $x^2+x-2=0$.
15. The maximum number of points of intersection of two non-intersecting squares and a circle is: (a) 4, (b) 8, (c) 16, (d) 7, (e) 12.
16. If the base of a triangle is fixed, the locus of the vertex for the triangle to have constant area is: (a) a circle whose diameter is the base, (b) one line parallel to the base, (c) two lines parallel to the base, (d) the perpendicular bisector of the base, (e) none of these answers.
17. If A is now 4 times as old as B, and in five years will be 3 times as old as B, the difference in their ages is: (a) 12 years, (b) 20 years, (c) 30 years, (d) 36 years, (e) 40 years.
18. If $\log_2 x=4\log_x 2$, then x equals: (a) 0 or 1, (b) 1 or -1, (c) 2 or $\frac{1}{2}$, (d) 4 or $\frac{1}{4}$, (e) impossible.
19. The ratio of a leg of an isosceles right triangle to the radius of the inscribed circle is: (a) $\sqrt{2}$, (b) $\sqrt{2}+1$, (c) $\sqrt{2}-1$, (d) $\sqrt{2}+2$, (e) $1/\sqrt{2}$.
20. If $y/x=3$, $y^x=x^y$, $x\neq 0$, then the value of x is: (a) ± 3, (b) $\pm\sqrt{3}$, (c) $\pm 3\sqrt{3}$, (d) $\pm\sqrt{3}/3$, (e) ± 1.
21. If a non-negative number x can be expressed in the form

$$1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cdots\cdots}}}$$

then the value of x is: (a) between 0 and $\frac{1}{2}$, (b) between $\frac{1}{2}$ and 1, (c) zero, (d) greater than one, (e) one.

22. Stock A costs \$10 per share, stock B \$3 per share, and stock C 50 cents per share. If a man had \$100 and wanted to buy 100 shares of stock with at least one share of each, how many shares of stock C would he buy? (a) one, (b) 50, (c) 76, (d) 90, (e) 94.
23. What is the product of the first and last terms of a geometric progression of four terms, whose middle terms are a and b? (a) a^2, (b) b^2, (c) a/b, (d) b/a, (e) ab.
24. Find the roots of: $2=-\sqrt{x-1}\sqrt{x+2}$ (a) 2 and -3, (b) 2, (c) -3, (d) no roots, (e) none of these answers.
25.



Angle b = angle d
Angle c = angle e
The value of angle x is: (a) c, (b) $b+c$, (c) $180^\circ-(b+c)$, (d) $180^\circ-2a$, (e) $a+c$.

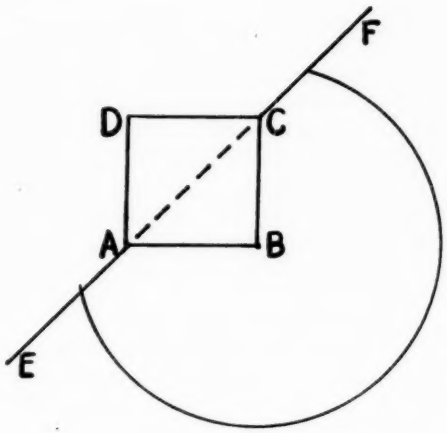
PART II (50%)—ONE HOUR

Show all work on this part. Partial credit will be given for partial solutions.

1. Solve for all values of x:

$$(2x^2-x-1)^2-8x^2+4x-8=0.$$

2.



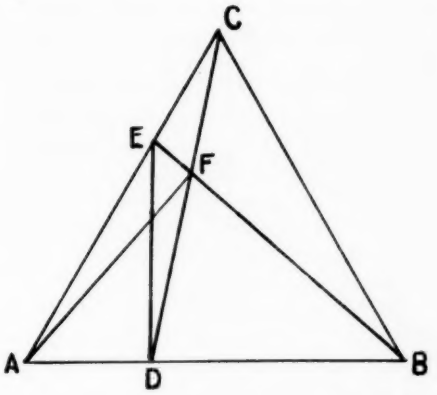
$ABCD$ is a square barn of edge 60 feet. AE and CF are fences along the extensions of diagonal AC. If a cow is tied to a rope $60\sqrt{2}$ feet long and the other end of the rope tied to corner B of the barn, find the area of the land over which the cow can graze.

3. Write the nth term of the series

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\cdots .$$

Find an expression for the sum of the first n terms of the series and prove it to be correct.

4.



Given: Triangle ABC is equilateral
$AE=2EC$, $DB=2AD$
Prove: (a) DE is perpendicular to AB
(b) AF is perpendicular to EB

How I teach algebra

Contributed by Mary White Duke, St. Anne's School, Charlottesville, Virginia.

For some time I have been concerned over the mathematical situation in our school (a private, college-preparatory school for girls) because too few of our students are producing a level of work commensurate with their ability. I have been studying myself, the students, their arithmetic background, various texts on arithmetic and algebra, and the methods suggested for the teaching of algebra in order to find what caused this unsatisfactory level of work. As a result I am now experimenting in each assigned group with some of my ideas. In explaining what I am doing, I shall have to digress at times to relate the reasons for what I am doing, in order to defend my ideas, methods, and objectives.

In the first place, I looked at myself as a teacher to discover what part of the trouble lay with me. I found my first directive on a short answer test when I noticed the word "t-o-w-e-l" written as the answer to a question involving "power." I pondered long over a possible explanation for this unexpected answer, and I realized my Southern accent had misled the student. Perhaps my explanation had not taken care of an obvious lack of acquaintance with the test, or perhaps a very limited knowledge of words had caused this student not to question what she had heard. I took immediate action, and now give a list of new mathematical terms at the beginning of each chapter, followed by a spelling lesson with correc-

tive exercises for misspelled words. To insure further familiarity with these words, I devise short answer tests that call for the specific words and count the question wrong if the word given is misspelled, for I maintain that a *bynomial* is not the same as a *binomial*. This is only my first step in emphasizing that mathematics is an exact science.

Having discovered that the student's lack of mathematical vocabulary was a contributing factor to the distressing mathematics situation, I examined my own way of presenting the subject matter. I read many texts for varied methods of attack in order to evolve one for myself. I studied definitions from the standpoint of the student who knew no mathematics and was seeking to learn. This study of myself and text led to requiring a notebook for algebra in which the student wrote simple, understandable definitions. I decided I had to take up a new term or an operation as though the student had never heard of it before and couch my sentences in the simplest possible language. I found, however, that it was necessary to check on what was taken down in the notebooks, for even then the students might not have heard what I said. After this check, I required each student to learn the definitions and next day to write her interpretation of each. I received a jumble of words which left all knowledge of grammar in the English room. So I had to take the time to explain that these pupils had a "funny" mathematics teacher who expected correct English, be it written or spoken, in her classroom. I required recopying until I was satisfied with the English and the mathematical content, and, although this was tedious and painful at the beginning, the girls soon "caught on" and even formulated good definitions occasionally on their own. I spent a great deal of time in making words meaningful, for I am convinced that I must make certain that the student has a thorough knowledge of the terms in order to prevent algebra from becoming a jumble of queer figures and letters that get more puzzling each day.

Then again I found in trying to formulate simple, understandable definitions that confusion often arises in the minds of the students because of the fact that words used in arithmetic are full of the inconsistencies of language and of process:

(a) What does 4 times 3 or any other part of the multiplication tables mean to the student?

(b) Why is "subtraction" used instead of "difference"?

(c) Why 12 divided by 4, instead of 12 divided into 4's or how many 4's in 12?

(d) Why reduce in connection with fractions and thus create the idea of a smaller fraction?

(e) Why "square inch" instead of "inch square"?

(f) Why "decimal" instead of "decimal fraction"?

(g) Why length times width equal area, thus leading to the idea of feet times feet give square feet?

(h) Why Case I, Case II, Case III of per cent instead of a continuation of the product idea?

I set about to clarify these and other obviously (to me) misleading terms in arithmetic and algebra by setting up both algebra and arithmetic upon the bases of only four fundamental operations with underlying principles for each process. I try to show the student that she can go from 5 times "what number" equals 20, to 5% times "what number" equals 20, to 5 times N equals 20. I point out that "simplify" in arithmetic and algebra refers to the same operation; that cost in dollars $+ 12 = \$40$ is the same as $C + 12 = 40$. I teach pupils to look for prime factors as a means of changing the form of fractions and as a means of finding common denominators and roots of numbers. I go from 5% times 6 to .05 times 6, to .05 times N. The three cases of per cent can all be explained as the equivalent of one quantity times another quantity equals a product, and I find the unknown quantity

of arithmetic in the same way I find the value of N.

Furthermore, believing strongly in my findings, I am centering my teaching on the following important ideas, and I insert one or more into each day's work:

(a) Addition, subtraction, multiplication, and division are at all levels fundamental operations of mathematics.

(b) Only like quantities can be added or subtracted, and their sum or difference is a like quantity, be it apples plus apples or 2 plus 2.

(c) One number times another number gives a product, be it 5 times 10 = 50, or ?% times 80 = 20, or $(x-y)(x+y)=x^2-y^2$. (Multiplication by 1 does not change the value, but it may change the form, since 1 is any number divided by itself.)

(d) A product divided by one of two factors gives the second factor of the product, be it long division, 0 divided by 12, or the factor theorem.

(e) A number may be expressed in many ways by changing its form without changing its value, as $\frac{1}{2}=2/4$; $a/b=ab/b^2$; or $-a/b=a/-b$.

(f) An equation is a balanced situation and must be kept balanced by performing one of the four fundamental operations on each member.

Along with these principles, I present each topic with the idea, "Is this step logical, and does it agree with the fundamental principles set forth?" I am trying to encourage pupils to consider the purpose of an operation before employing it. I teach no short cuts, but require pupils to know each step. Later, if a student discovers some shortcut and asks me privately about it, she may get my approval to use it. I never teach such rules as: "In order to subtract change the sign of the subtrahend and add," or "Transpose a term from one side of an equation to the other by changing the sign." I believe knowledge of the "why" is important and will eventually take care of the time element.

In developing each new topic logically and slowly, I find that constant, simple, short checks are necessary. Almost daily I give some type of short test to check and recheck the student and myself. I send several students to the board to work the same example, so I can watch for the type of error, while those remaining at their desks are working with the idea that we can learn from each other. I give short, to-the-point assignments to do out of class, in order for the student to discover, and for me to discover, if there is still something she does not understand. I find that I get honest cooperation, as a copied paper serves no purpose. Her grade comes from a subsequent test on this home assignment.

My work with the mechanics of algebra naturally led into a study of difficulties with verbal problems. After much experimentation, I have come to the conclusion that there are three contributing factors to the difficulty pupils have with verbal problems—poor reading ability, a preconceived idea that word problems are hard, and the refusal to put facts on paper. I am trying to combat these obstacles in the following ways:

(a) I drill in putting into algebraic shorthand such simple expressions as "N decreased by 5," "8 subtracted from N," "11 exceeds N by 2"; algebraic symbols for these and many such words having been put into a chart, copied into the notebook, and learned.

(b) I give a set-up for problems by type and these are recorded and learned.

(c) I put only a part of a problem on the board, as: If a two-digit number. . . and each writes u = unit's digit, t = ten's digit, and $10t+u$ = the number.

(d) I gradually increase the words in (c) until a whole problem is on the board. I give problems which pupils set up involving fractions and quadratics long before they have mastered the mechanics of handling them. This goes on until after Christmas, when they begin to complete a problem, for I tell them it is useless to try to solve a problem before they have sorted the information correctly. After I

am satisfied that there is a reasonable chance of success, I include one or two problems in each day's assignment.

In teaching I have to bear in mind that I am preparing students only for college, and of course that involves College Boards and Educational Records Bureau tests. Therefore, there is little or no place for mechanical mathematics. I have to devise means to present the subject matter in more than one way. I give multiple choice, true-false, and completion tests, twisting the material in every possible way. I try to test the understanding of a skill with questions that imply mechanics but need some thought to sort out, as: x^2+ax+b is exactly divisible by $(x+3)$ and $(x-4)$, then $a=?$, $b=?$, or ax^b-3 is a factor of $16x^2-9$, $a=?$, $b=?$. I use a workbook containing a review for College Boards and the New York Regents to give an outsider's view of the subject matter. I search daily for new ideas and welcome any new twist to the old.

In summary:

I. My aims are:

(a) To leave with the student the idea that good English is necessary in the mathematics room as well as in the English room, and that words should be correctly spelled.

(b) To develop a pride in neat, accurate work, honestly done for the purpose of learning.

(c) To increase the student's vocabulary and to teach the proper way to define a term.

(d) To make the student aware of mathematics as a logical subject and to train her to test her way with logic.

(d) To make mathematics fun and interesting and a challenge to the student's mind.

(f) To teach each girl to read and understand a problem, to sort out facts, and to handle them as the words suggest.

II. My methods are:

(a) To create an understanding of terms.

(b) To develop each step slowly, accurately, and logically.

(c) In some way to test understanding almost daily and to reteach when necessary.

(d) To eliminate special methods and to stick to a few sound, fundamental principles.

(e) To keep in mind that I am dealing with someone's child and to do for her what I would want done for mine.

Editor's Note: The editors of the department do not necessarily agree with all points of view, procedures, and methods reported in the various contributions. We believe that teachers can profitably read about what others are doing in their classes, even though they may decide that their own methods are preferable for their situations. A self-examination, such as that which Mrs. Duke imposed upon herself, would be valuable for all of us as teachers.

The New England Mathematics Institute

The Sixth Annual Institute for Teachers of Mathematics sponsored by the Association of Teachers of Mathematics in New England will be held August 19–26, 1954, at the Massachusetts Institute of Technology, Cambridge, Mass. Here is an unusual opportunity for a stimulating week of work, recreation, and inspiration, along the historic Charles River in Greater Boston with accommodations such as you could not purchase for double the price if you came "on your own."

There will be two general meetings daily, one in the morning, the other in the evening, addressed by outstanding mathematicians, and by leaders in the field of education. Discussion groups and laboratories are being planned for teachers working in the elementary, secondary, and college levels. Members of the Institute can enroll for three so-called group meetings each day.

Watch succeeding issues of this magazine for further information.—*M. Isabelle Savides, General Chairman.*

NCTM

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Thirty-second Annual Meeting

The Sheraton-Gibson Hotel, Cincinnati, Ohio
April 20–24, 1954

HOST ORGANIZATIONS
Mathematics Club of Greater Cincinnati,
Ohio Council of Teachers of Mathematics

THEME

Mathematics on the March

Tuesday, April 20, 1954

1:30–10:00 P.M. Meeting of the Board of Directors

Wednesday, April 21, 1954

8:30 A.M.–10:00 P.M. Meeting of the Board of Directors

9:00 A.M.–4:00 P.M. Visits to Cincinnati schools

3:30 P.M.–9:00 P.M. Registration

Thursday, April 22, 1954

8:00 A.M.–5:00 P.M. Registration

8:00–11:00 A.M. **Fifth Annual Delegate Assembly.** Meeting of the Official Delegates of the Affiliated Groups.

Presiding: MARY C. ROGERS, Roosevelt Junior High School, Westfield, N. J.

The second session of the Delegate Assembly is a dinner session, Thursday evening.

9:00–9:30 A.M. Senior High School Demonstration Lesson by Television, Cincinnati Public Schools

9:00 A.M.–5:00 P.M. School and commercial exhibits

9:00 A.M.–12:00 M. Visit to industries of Cincinnati

9:00 A.M.–12:00 M. Tour of Cincinnati

10:30 A.M.–12:00 M. Projection of mathematics films

11:00 A.M.–12:00 M. Meeting of the State Representatives

Presiding: MYRL H. AHRENDT, Executive Secretary, The National Council of Teachers of Mathematics, Washington, D. C.

11:00 A.M.–2:15 P.M. Committee Meetings

The schedule for committee meetings will be listed in the Official Program.

2:30–4:30 P.M. **Research Section**

The Section is sponsored by the National Council Committee on Research

Presiding: HOWARD F. FEHR, Teachers College, Columbia University, New York, N. Y.

The Work of the Research Committee of the Council

HENRY VAN ENGEN, Iowa State Teachers College, Cedar Falls

The Research in Mathematical Education in 1952

KENNETH E. BROWN, Specialist for Mathematics, U. S. Office of Education, Washington, D. C.

Types of Rationalization in Teaching Arithmetic

GEORGE MCMEEN, State Teachers College, Newark, N. J.

The Preparation of College Mathematics Instructors

LYMAN C. PECK, Iowa State Teachers College, Cedar Falls

Significant Research Now Under Way

MAURICE HARTUNG, University of Chicago, Chicago, Ill.

NATHAN LAZAR, Ohio State University, Columbus, Ohio

JOHN KINSELLA, New York University, New York, N. Y.

MYRON ROSSKOPF, Teachers College, Columbia University, New York, N. Y.

F. LYNWOOD WREN, George Peabody College for Teachers, Nashville, Tenn.

2:30–4:30 P.M. **College Section**

Ferment in Freshman Mathematics, ROTHWELL STEPHENS, Knox College, Galesburg, Ill.

A Glimpse at the Calculus, EUGENE W. HELLMICH, Northern Illinois State Teachers College, De Kalb

Topics from Modern Algebra That Can Be Adapted to College Freshmen, KENNETH O. MAY, Carleton College, Northfield, Minn.

2:30–4:00 P.M. **Mathematics in Industry Section**

My Experiences on a Ford Foundation Fellowship, SARAH AUSTIN FROST, Greenville High School, Greenville, S. C.

The Engineer in Industry, EVERITT S. LEE, Manager, Technical Public Relations, General Electric Company, Schenectady, N. Y.

Mathematics and the Technical Manpower Shortage, LEONARD G. YODER, Yoder Instruments, Inc., East Palatine, Ohio

2:30–4:30 P.M. **Elementary School Laboratory**

Grades 1–3, Leader: SALLY GRIFFITH, Schiel School, Cincinnati, Ohio

Grades 4–6, Leader: CARRIE MOUNT, Whittier School, Cincinnati, Ohio

2:30–4:30 P.M. **Junior High School Laboratory**

Laboratory Techniques and Procedures in Junior High School Geometry

Leader: FRANCES M. BURNS, Oneida High School, Oneida, N. Y.

2:30–4:30 P.M. **Senior High School Laboratory**

Leaders: HOPE CHIPMAN, University High School, Ann Arbor, Mich.

GERTRUDE C. HAZZARD, Guilford High School, Guilford, Conn.

2:30–3:45 P.M. **Elementary School Section**

Making Pupils Aware of the Significance of Arithmetic in Their Daily Lives, HELEN ALLPIN, Department of Instruction, Denver Public Schools, Denver, Colo.

The Place of Drill in Achieving Functional Competence in Arithmetic, RALPH J. COOKE, Director of Elementary Education, Fond du Lac, Wis.

2:30–3:45 P.M. **Junior High School Section**

Some Important Understandings in the Mathematics of Grades Seven and Eight, HELEN A. SCHNEIDER, Oak Junior High School, La Grange, Ill.

Important Facts About Fractions, HUMPHREY C. JACKSON, Charles A. Parcella Junior High School, Grosse Pointe, Mich.

2:30–3:45 P.M. **Senior High School Section,**

Panel Discussion: *What Mathematics Should We Teach in the 12th Year*

FRANCIS G. LANKFORD, JR., University of Virginia, Charlottesville, Chairman

R. A. BAUMGARTNER, High School, Freeport, Ill.

ENOCH BURTON, Shortridge High School, Indianapolis, Ind.

DAVID H. FRANK, Forest Hills High School, Forest Hills, N. Y.

FRANKLIN FREY, Cass Technical High School, Detroit, Mich.

MARGARET JOSEPH, Shorewood High School, Milwaukee, Wis.

ALICE M. REEVE, South Side High School, Rockville Centre, N. Y.

4:00–5:15 P.M. **Elementary School Section**

Utilizing Dramatization and Visual Representation to Improve Problem Solving Ability, GLADYS TATE, Gridley Junior High School, Erie, Pa.

The Effective Utilization of Diagnostic Procedures in Arithmetic, CALVIN H. REED, University of Nebraska, Lincoln

4:00–5:15 P.M. **Secondary School Section**

Helping Junior High School Pupils Explore Mathematics, DONALD W. LENTZ, Parma Schaaf High School, Parma, Ohio

Concepts from Arithmetic Which May Be Used in Teaching Fractions in Algebra, EDITH TRUENFELS, Putney School, Putney, Vt.

4:00–5:15 P.M. **Discussion Groups**

A. *Making Fractions and Decimals Meaningful to Fifth and Sixth Graders*, MILDRED COLE, C. M. Bardwell School, Aurora, Ill.

B. *Revitalizing Our Algebra Course at the Junior High Level*, DOROTHY BRAGONIER, Marshall College, Huntington, W. Va.

C. *How To Vary Geometry Classroom Procedures*, HAZEL DULING, East High School, Madison, Wis.

D. *Business and Education: The Coordinates of a Strong Community*, RICHARD M. JOHNSTON, Woodrow Wilson Junior High School, Tulsa, Okla.

E. *Is Your General Mathematics Course Really "General"?* GEORGE BULLIS, State College, Platteville, Wis.

5:15–7:45 P.M. Second Session of the Fifth Delegate Assembly, Rookwood Room, Hotel Sinton

A continuation of the Thursday morning session of the Official Delegates of the Affiliated Groups. This will be a dinner session.

8:00 P.M. **General Session**

Welcome to the Convention, CLAUDE V. COURTER, Superintendent of Schools, Cincinnati, Ohio

Teaching and Learning in a Good School, DONALD P. COTTRELL, Dean, College of Education, Ohio State University, Columbus

Presentation of the Twenty-Second Yearbook, JOHN KINSELLA, New York University, New York; JOY MAHACHEK, State Teachers College, Indiana, Pa.; PHILIP PEAK, Indiana University, Bloomington, Ind.; VERYL SCHULT, Willson Teachers College, Washington, D. C.; *Chmn.:* JOHN R. CLARK, Lahaska, Pa.

9:30 P.M. **Reception**

Hosts: Mathematics Club of Greater Cincinnati, Ohio Council of Teachers of Mathematics

Friday, April 23, 1954

7:15 A.M. Breakfast for State Representatives

7:15 A.M. Reeve Gang Breakfast, Hotel Coffee Shop

8:00 A.M.–5:00 P.M. Registration

9:00–9:30 A.M. Junior High School

Demonstration lesson by television, Cincinnati Public Schools

9:00 A.M.–5:00 P.M. School and commercial exhibits

9:40–10:40 A.M. **General Session**

To Be Effective, Guidance and Counseling Programs in Mathematics Must Be Continuous and Comprehensive Through the Grades, C. L. THIELE, Director of Exact Sciences, Public Schools, Detroit, Mich.

10:50 A.M.–12:15 P.M. **Elementary School Section**

The Basic Understandings, Concepts, and Facts in an Arithmetic in the Elementary Grades, VINCENT J. GLENNON, Syracuse University, Syracuse, N. Y.

Developing Confident, Self-reliant Learners in Arithmetic, ELINOR B. FLAGG, Illinois State Normal University, Normal

Values of Pupil-Teacher Planning in Arithmetic, L. H. WHITCRAFT, Ball State Teachers College, Muncie, Ind.

10:50 A.M.–12:15 P.M. **Junior High School Section**

Practical Mathematics Is Challenging to Students, FRANCIS G. LANKFORD, JR., University of Virginia, Charlottesville

Determining Algebraic Ability Through Eighth-Grade Mathematics, LELA COBB, Junior-Senior High School, Wellington, Kan.

Techniques for Evaluating the Effectiveness of Group Dynamics for the Gifted Student, HUBERT B. RISINGER, Davey Junior High School, East Orange, N. J.

10:50 A.M.–12:15 P.M. **Secondary School Section**

How Children Learn Geometry, R. R. SMITH, Springfield Public Schools, Springfield, Mass.

What Do We Want Our Students to Learn in Geometry, ROBERT L. PINGRY, University of Illinois, Urbana, Ill.

10:50 A.M.–12:15 P.M. **Individual Differences Section**

Provisions by Our Schools for Fast and/or Slow Learners, KENNETH E. BROWN, Specialist for Mathematics, U. S. Office of Education, Washington, D. C.

Teaching Different Ability Levels in the Same Class, JOYCE BENBROOK, University of Houston, Houston, Tex.

Is There a Democratic Way of Teaching Arithmetic? ROBERT S. FOUCH, Arizona State College, Tempe

10:50 A.M.–12:15 P.M. **Topics from Mathematics Section**

Elementary Notions in Relativity Theory, J. SUTHERLAND FRAME, Michigan State College, East Lansing, Mich.

An Elementary Theorem from Arithmetic, ALFRED PUTNAM, University of Chicago, Chicago, Ill.

10:50 A.M.–12:15 P.M. **General Mathematics Section**

Panel Discussion: *What Constitutes a Good General Mathematics Program?* LEE E. BOYER, State Teachers College, Millersville, Pa., Chairman

ONA KRAFT, Collinwood High School, Cleveland, Ohio

L. W. LAVENGOOD, Mathematics Supervisor, Tulsa, Okla.

HENRY A. MEYER, Central High School, Evansville, Ind.

MARY S. MOREHART, Everett Junior High School, Columbus, Ohio

H. VERNON PRICE, State University of Iowa, Iowa City

10:50 A.M.–12:15 P.M. **Correspondence Study and Adult Education Section**

Opportunities Through Correspondence Study in Mathematics, JOHN A. BROWN, University of Wisconsin, Madison

Characteristics of a Good Correspondence Study Mathematics Course, PHILIP PEAK, Indiana University, Bloomington

Elementary Mathematics in Adult Education, WILLIAM SCHUTZMAN, New York University, New York, N. Y.

10:50 A.M.–12:15 P.M. **College Section**

Can We Improve the Offerings in Analytic Geometry? MILDRED TAYLOR, Mary Baldwin College, Staunton, Va.

Special Mathematics Service Courses, W. C. MCDANIEL, Southern Illinois University, Carbondale

12:30–2:15 P.M. Committee Meetings

The schedule of committee meetings will be included in the Official Program.

2:30–4:30 P.M. **Affiliated Groups Section**

Co-relationship of Science and Mathematics, KENNETH E. VORDENBERG, Assistant Supervisor of High School Science, Cincinnati Public Schools, Cincinnati, Ohio

Mathematics—A Language, GEORGE R. SEIDEL, Research Supervisor of E. I. DuPont de Nemours and Company, Wilmington, Del.

From Abacus to Univac, FRED G. FENDER, Rutgers University, New Brunswick, N. J.

2:30–4:30 P.M. **Elementary School Section**

Symposium: *Curriculum Trends in the Teaching of Arithmetic in the Elementary School*

Chairman and Discussion Leader: ESTHER HOELDTKE, Public Schools, Buffalo, N. Y.

Participants: MILDRED STONE, State Teachers College, Salem, Mass.

L. W. LAVENGOOD, Tulsa Public Schools, Tulsa, Okla.

BESS TOWNSEND, North Texas State College, Denton, Tex.

2:30–4:30 P.M. **Section Sponsored by the Ohio Council of Teachers of Mathematics**

Presiding: H. C. CHRISTOFFERSON, Miami University, Oxford, Ohio

Creative Teaching of Mathematics

Analyst: HAROLD P. FAWCETT, Ohio State University, Columbus, Ohio

Panel: OSCAR SCHAAF, Ohio State University, Columbus

JOHN SCHACHT, Bexley High School, Bexley, Ohio

EMMALOU BRUMFIELD, Kent State University, Kent, Ohio

WILLIAM LOWRY, Marion-Franklin High School, Columbus, Ohio

Note: There will be a short business meeting of the Ohio Council following this session.

2:30–4:30 P.M. **Teacher Education Section**

The Use of Research Findings in the Preparation of Mathematics Teachers, JOHN J. KINSELLA, New York University, N. Y.

What Can the College Teacher Do To Improve Teacher Education? HARLAN MILLER, Texas State College for Women, Denton

The Long Road from Reality to Symbols, NATHAN LAZAR, Ohio State University, Columbus

2:30–3:45 P.M. **Elementary School Section**

Building an Understanding of the Structure of Our Number System, J. FRED WEAVER, Boston University, Boston, Mass.

Long Division—A Trouble Spot in the Teaching and Learning of Arithmetic, KENNETH G. FULLER, Teachers College of Connecticut, New Britain

2:30–3:45 P.M. **Junior High School Section**

A Junior High School Workshop for Mathematics Teachers, MARY A. POTTER, Consultant in Mathematics, Public Schools, Racine, Wis.

Taking the Drudgery Out of Problem Solving, ALICE M. HACH, Slauson School, Ann Arbor, Mich.

Developmental Mathematics in the New York City Schools, LAURA K. EADS, Bureau of Curriculum Research, Board of Education, New York, N. Y.

2:30–3:45 P.M. **Algebra Section**

Number Systems, JAMES A. WARD, University of Kentucky, Lexington

Algebra with Fewer Rules and Better Reasons, WILLIAM R. RANSOM, Tufts College, Medford, Mass.

2:30–3:45 P.M. **Section on Mathematics for Elementary Teachers**

Mathematics as a Part of General Education for Elementary Teachers, LYLE DIXON, Arkansas State College, State College

A Special Course in Mathematics for Elementary Teachers, RALPH HUFFER, Beloit College, Beloit, Wis.

Casting Out Nines and Elevens, HAROLD D. LARSEN, Albion College, Albion, Mich.

4:00–5:15 P.M. **Elementary and Junior High School Section**

Content and Goals in Arithmetic Education in the Elementary School (with emphasis on grades five and six), BEN A. SUELTZ, State Teachers College, Cortland, N. Y.

Closer Articulation Between the Elementary and the Junior High School, HERSCHEL E. GRIME, Directing Supervisor of Mathematics, Public Schools, Cleveland, Ohio

4:00–5:15 P.M. **Secondary School Section**

Arithmetic in the Junior High School, FOSTER E. GROSSNICKLE, State

Teachers College, Jersey City, N. J.

The Personal Interview, a Helpful Diagnostic Procedure, Joseph N. Payne, Indian Springs School, Helena, Ala.

4:00–5:15 P.M. **Discussion Groups**

F. *Beginning Arithmetic, Some Whats and Hows*, Mary Samter, Central Wisconsin State College, Stevens Point, Wis.

G. *How Should We Introduce Coordinate Geometry into the Tenth Year Program?* Alice L. Griswold, Garden City High School, Garden City, N. Y.

H. *High School Trigonometry for College Preparatory Students*, Bess Patton, High School, Atlanta, Ga.

I. *Concepts Which Should Be Stressed in Algebra*, Annie John Williams, Durham High School, Durham, N. C.

J. *What Mathematics Teachers at All Levels Can Do To Meet the Growing Demand for More and Better Mathematics and More and Better Mathematics Students*, James H. Zant, Oklahoma A & M College, Stillwater

4:00–5:15 P.M. Projection of Mathematics Films

7:00 P.M. **Banquet**

Address: *Through Europe with Slide Rule and Camera*, Henry W. Syer, Boston University, Boston, Mass.

Saturday, April 24, 1954

8:00–9:00 A.M. Annual Business Meeting

8:00–12:00 M. Registration

9:00–12:00 M. School and commercial exhibits

9:00–9:30 A.M. Primary and Intermediate Grades

Demonstration Lesson by Television, Cincinnati Public Schools

9:40–10:40 A.M. **General Session**

Rate of Progress in Learning Arithmetic, Esther J. Swenson, University of Alabama, University, Ala.

10:50 A.M.–12:15 P.M. **Kindergarten and Primary Grades Section**

A Systematic Approach to Arithmetic in Kindergarten, Sara A. Rhue, University of Wisconsin, Madison

The Development of the Concept of Time, Louise Beltramo, State University of Iowa, Iowa City

10:50 A.M.–12:15 P.M. **Elementary School Section**

Contributions of Work with Measures to the Development of Functional Competence in Arithmetic, Lois Knowles, University of Missouri, Columbia, Mo.

Unifying Ideas in the Arithmetic of the Elementary School, H. G. Wheat, West Virginia University, Morgantown, W. Va.

10:50 A.M.–12:15 P.M. **Junior High School Section**

Developing Mathematical Literacy in the Youth of America, Milton W. Beckman, Teachers College High School, Lincoln, Neb.

Seeing What They Read, Glenadine Gibb, Iowa State Teachers College, Cedar Falls

Content and Goals of Junior High School Geometry, Margaret M. Dunn, Bloomfield Junior High School, Bloomfield, N. J.

10:50 A.M.–12:15 P.M. **High School and College Section**

Panel Discussion: *Better Articulation Between High School and College*, Analysts: H. Vernon Price, State University of Iowa, Iowa City

H. S. Kaltenborn, Memphis State College, Memphis, Tenn.

Daniel W. Snader, University of Illinois, Urbana

Harry W. Charlesworth, East High School, Denver, Colo.

Sheldon Myers, Ashville High School, Ashville, Ohio

10:50 A.M.–12:15 P.M. **Section on Topics from Geometry**

The Role of the Fifth Postulate, Mildred Taylor, Mary Baldwin College, Staunton, Va.

Closely Packed Spheres, Walter B. Carver, Cornell University, Ithaca, N. Y.

10:50 A.M.–12:15 P.M. **Supervisors Section**

Panel Discussion: *Better Supervision of Mathematics Teaching*, MARY A. POTTER, Consultant in Mathematics, Racine, Wis., Chairman.

HERSCHEL GRIME, Mathematics Supervisor, Cleveland, Ohio.

GERALDINE KAUFFMAN, Supervisor of Mathematics and Science, East Chicago, Ill.

L. W. LAVENGOOD, Mathematics Supervisor, Tulsa, Okla.

EUGENE F. PECKMAN, Senior Supervisor, Science and Mathematics, Pittsburgh, Pa.

LOUIS F. SCHOLL, Supervisor of Mathematics, Buffalo, N. Y.

VERYL SCHULT, Director of Mathematics, Washington, D. C.

10:50 A.M.–12:15 P.M. **Section on Second Year of Algebra**

The History of the Binomial Theorem, VERA SANFORD, State Teachers College, Oneonta, N. Y.

Some Teaching Aids That May Be Used in Second Year Algebra, EMIL BERGER, Monroe High School, St. Paul, Minn.

10:50 A.M.–12:15 P.M. **Contests Section**

This section is sponsored by the National Council Committee on Contests, Scholarships, and Talent Search.

What Mathematics Contests Can Contribute

Panel Members: WALTER H. CARNAHAN, Purdue University, Lafayette, Ind., Chairman

HOUSTON KARNES, Louisiana State University, Baton Rouge

W. R. KRICKENBERGER, Arsenal Technical High School, Indianapolis, Ind.

DANIEL LLOYD, Wilson Teachers College, Washington, D. C.

12:30 P.M. Luncheon

2:30–4:00 P.M. **Elementary School Section**

Factors Which Increase Accuracy in Computation in Arithmetic, ROBERT H. KOENKER, Ball State Teachers College, Muncie, Ind.

Classroom Activities Which Develop and Reinforce Meanings in Arithmetic, ADALINE P. HAGAMAN, Somerset County Helping Teacher, Plainfield, N. J.

Challenging the Rapid Learner in Our Arithmetic Program, ANTHONY SCHWARZ, Butler University, Indianapolis, Ind.

2:30–4:00 P.M. **Junior High School Section**

The Problems We Have Always with Us, EDITH WOOLSEY, Sanford Junior High School, Minneapolis, Minn.

What Gains Should Students Expect to Make in Junior High School Mathematics, DOROTHY SWARD, Roosevelt Junior High School, Beloit, Wis.

Projects in Seventh and Eighth Grade Classes, DORIS L. MCLENNAN, Theodore Roosevelt Junior High School, Syracuse, N. Y.

2:30–4:00 P.M. **Accelerated Program Section**

This section is sponsored by The West Virginia Council of Mathematics Teachers. Panel Discussion: *Kalgometrics—A Report of a Three-Year Project in Giving to "Able" Youth an Accelerated Mathematics Program.* Presiding: MARGARET F. HOLMES, High School, Dunbar, W. Va.; Moderator: ANDREW V. KOZAK, Concord College, Athens, W. Va. Students in the project will participate in the discussion.

2:30–4:00 P.M. **Senior High School Section**

What Algebra Should Be Offered in the Third Year, ELSIE PARKER JOHNSON, Oak Park High School, Oak Park, Ill.

Bulletin Boards, One Means of Stimulating Interest in Mathematics, WALTER H. CARNAHAN, Purdue University, Lafayette, Ind.

Problem Solving as an Approach to Teaching General Mathematics in the Senior High School, Barnett Rich, Richmond Hill High School, New York, N. Y.

2:30–4:00 P.M. **College and Teacher Education Section**

Report on the Conference on Collegiate Mathematics at Boulder, Summer 1953

Analysts: J. A. COOLEY, University of Tennessee, Knoxville

RUTH B. SMYTH, The College of Wooster, Wooster, Ohio

R. A. ROWLAND, Ohio Wesleyan University, Delaware

5:00–5:30 P.M. Review of the Convention (Television)

Report to Cincinnati.

Participants: JOHN R. MAYOR, University of Wisconsin, Madison

H. GLENN AYRE, Western Illinois State College, Macomb

MARY C. ROGERS, Roosevelt Junior High School, Westfield, N. J.

IRENE SAUBLE, Director of Exact Sciences, Detroit, Mich.

MARIE S. WILCOX, Thomas Carr Howe High School, Indianapolis, Ind.

7:00 P.M. Meeting of the Board of Directors

Announcements

Registration

The registration fee is fifty cents for members of The National Council of Teachers of Mathematics, members of the Mathematical Association of America, and for teachers in elementary schools. The fee for non-members and visitors is $1.50. Undergraduate students sponsored by a faculty member, relatives of members, invited speakers who are not members, and members of the press are not charged the registration fee but should register. You are urged to register in advance and requested to check in at the Registration Desk on the Ball Room Floor upon your arrival at the meeting. Use the Advance Registration and Reservation Form to be found on page 155. Registrations and reservations received before April 1 will be acknowledged by return mail. Inquiries concerning registration should be addressed to Miss Joan Bollenbacher, Cincinnati Board of Education, McMillan Street, Cincinnati 6, Ohio.

Room Reservations at the Sheraton-Gibson

All Rooms with Bath and Radio; 700 with T.V.; 900 with air conditioning
Single Rooms for One—$5.35 and up
Double Rooms for Two—$7.85 and up
Twin Beds for Two—$9.35 and up
Parlor Suites (2 rooms)—$21.35 and up
(Information regarding special dormitory rates upon request).
Requests for reservations should be sent directly to The Sheraton-Gibson Hotel, Cincinnati 1, Ohio

Transportation

Cincinnati is easily accessible by public or private transportation. The Sheraton-Gibson Hotel is located on Walnut Street between Fourth and Fifth Streets. (Walnut Street is one-way, south.) The hotel may be reached from the Union Terminal via city bus or by taxi. Limousine service is available from the Greater Cincinnati Airport (located in Boone County, Kentucky) to the downtown hotels.

Banquet and luncheon reservations

The Convention banquet will be held on Friday night and the luncheon on Saturday as is customary. Reservations for the banquet and luncheon should be made in advance, accompanied by check or money order. All orders received before April 1 will be acknowledged by return mail. Banquet $3.50; Luncheon $2.75, tips included. Use the Advance Registration and Reservation Form. Banquet tickets must be purchased by noon on Friday and luncheon tickets by 8:00 P.M. on Friday.

Visiting schools of Greater Cincinnati

Visitors are cordially invited to observe work in schools in the various systems in Greater Cincinnati on Wednesday. The Cincinnati Public Schools, Hamilton County Public Schools, and Cincinnati Parochial Schools offer opportunities to observe classes in mathematics in elementary, junior high, and senior high schools. Schedules will be available at the Information Desk, but requests to observe should be mailed in advance to Miss F. Jane Ragland, Cincinnati Board of Education, McMillan Street, Cincinnati, Ohio.

Mathematics laboratories

Three mathematics laboratories are planned at the elementary, junior high school, and senior high school levels. The plan of operation will include a brief talk and demonstration of the construction and use of models in the classroom. Each member will be given an opportunity to make one or more models for which the minimum charge for materials used will be made.

Advance registration is requested in order that sufficient materials may be supplied to provide for all participants.

Supplies and equipment

Speakers and other participants on the program who need blackboards, projection equipment, or other materials should communicate with Mr. Kenneth Vordenberg, Cincinnati Board of Education, McMillan Street, Cincinnati 6, Ohio, before April 1.

Commercial exhibits

Textbooks and commercial teaching aids will be exhibited in the Ball Room Foyer from Thursday morning at 9:00 to noon on Saturday. Inquiries for exhibit space should be addressed to Dr. H. C. Christofferson, Miami University, Oxford, Ohio.

School exhibits

There will be an exhibit of mathematics classroom materials and non-commercial teaching aids in the Ball Room Foyer and on the Mezzanine from Thursday morning at 9:00 to Saturday noon. Teachers who are willing to exhibit their materials are requested to communciate with Mr. Howard Luedeke, Walnut Hills High School, Cincinnati 7, Ohio.

Meals for special groups

If any group wishes to arrange for a dinner on Thursday, a breakfast on Thursday, Friday or Saturday, or a luncheon on Friday, the local committee will help make arrangements. Please write to Miss Anna Marie Evans, Cincinnati Board of Education, McMillan Street, Cincinnati 6, Ohio.

Location of meeting rooms

The Sheraton-Gibson Hotel is headquarters for the Convention. All rooms and parlors used for the meetings and discussions are located in this hotel.

Mail and telegrams

Mail and telegrams for those attending the Convention should be addressed in care of The National Council of Teachers of Mathematics, The Sheraton-Gibson Hotel, Cincinnati 1, Ohio. Mail may be obtained at the convention registration desk on the Ball Room Floor.

Certificate of Attendance

If you care to take back to your school authorities a statement certifying your attendance at the Convention, make a request for it at the Registration Desk on Saturday morning.

Program Committee

John R. Mayor, Madison, Wisconsin, Chairman
H. Glenn Ayre, Macomb, Illinois
Mary C. Rogers, Westfield, New Jersey
Irene Sauble, Detroit, Michigan
Marie S. Wilcox, Indianapolis, Indiana

Local Committee Chairmen

General Chairman: Mildred Keiffer, Cincinnati Public Schools, Cincinnati, Ohio
Arrangements: Kenneth Vordenberg, Cincinnati Public Schools, Cincinnati, Ohio
Banquet: Helen Walter, Western Hills High School, Cincinnati, Ohio
Meals Coordinator (Special Meals): Anna Marie Evans, Cincinnati Public Schools, Cincinnati, Ohio
Commercial Exhibits (Space): H. C. Christofferson, Miami University, Oxford, Ohio
Commercial Exhibits (Local): Donald Drake, Kennedy-Silverton School, Cincinnati, Ohio
Daily Newsletter: Robert C. Price, University of Cincinnati, Cincinnati, Ohio
Films and Filmstrips: Dorothea Edgeworth, Western Hills High School, Cincinnati, Ohio; Edwin Sutton, Western Hills High School, Cincinnati, Ohio
Finance: Marie Becker, Walnut Hills High School, Cincinnati, Ohio; Norma Struke, Rothenberg Junior High School, Cincinnati, Ohio
Hospitality: Eleanor Graham, Withrow High School, Cincinnati, Ohio
Industrial Visit: Kenneth Scheurer, Walnut Hills High School, Cincinnati, Ohio
Information: Nanabel McNelly, Oyler School, Cincinnati, Ohio
Luncheon: Ruth Austin, Hughes High School, Cincinnati, Ohio
N.C.T.M. Membership: James V. Naugle, Wyoming High School, Wyoming, Ohio
Publicity and Promotion: Svet Nankovitch, South Avondale School, Cincinnati, Ohio
Reception: Dorothy Beaver, Bloom Junior High School, Cincinnati, Ohio
Registration: Joan Bollenbacher, Cincinnati Public Schools, Cincinnati, Ohio
School Exhibits: Howard Luedeke, Walnut Hills High School, Cincinnati, Ohio; and James K. Koger, Hamilton High School, Hamilton, Ohio
School Visits: Fannie Jane Ragland, Cincinnati Public Schools, Cincinnati, Ohio
Signs and Printing: C. O. Tower, Central High School, Cincinnati, Ohio

The Fifth Delegate Assembly

by Daniel B. Lloyd, Wilson Teachers College, Washington, D. C.

The *Fifth Delegate Assembly* of the National Council of Teachers of Mathematics will be held at Hotel Sheraton-Gibson, in Cincinnati, Ohio, on Thursday, April 22, 1954. This is the opening morning of the three-day annual meeting of the Council. The entire program of the Delegate Assembly will be completed on that first day, starting with a three-hour session, 8 to 11 A.M., and closing with a dinner program from 5:30 to 7:30 P.M. This will avoid conflicts with other scheduled meetings and thus do "the greatest good for the greatest number."

Every Affiliated Group is not only asked, but urged, to send at least one delegate to these sessions. If each delegate will become acquainted with the topics listed below on the program, and be ready to bring to our assembly some vital and illuminating suggestions on one or more of them, much good will be done of permanent and professional value. We might recall the old adage that "In order to bring back the wealth of the Indies, we must take the wealth of the Indies with us."

It is indubitable that much of the present effective planning of the National Council has been influenced by the important thinking emanating from this body of national representatives. The growing importance of the Delegate Assembly has been recognized not only by the attending delegates sent by their respective affiliated groups throughout the country to these annual meetings, but also by the national officers, specialists, speakers who have addressed the body, and by visitors who have listened to its deliberations.

To maintain and increase the prestige of this Assembly, the program this year has been planned with a view to allowing a maximum opportunity for expression and interchange of constructive ideas among delegates. Oral reports will be limited and furnished to the delegates in outline form. Each report will be followed by questions, comments from the floor, and free, though we hope, brief and pertinent, discussion. There will also be ample opportunity for any delegate to introduce new and creative items to improve the functioning of the National Council and the component groups of the Council.

The tentative agenda outlined below is based on helpful ideas from many members throughout the country. However, there may necessarily be certain revisions between now and April. It is hoped that every reader will offer constructive suggestions. The following members, in addition to the writer who is the chairman, constitute the Planning Committee for this year's assembly: Charlotte Carlton, Miami Beach, Florida; Mary Lee Foster, Arkadelphia, Arkansas; Robert S. Fouch, Arizona State College; Ruth Lee Green, Wilmington, Delaware; Geraldine Kauffman, East Chicago, Indiana; and Dorothy Sward, Beloit, Wisconsin.

The Regional Representatives of the Affiliated Groups also served as ex-officio members of the committee. They are:

Northeast—Jackson B. Adkins, Phillips Exeter Academy, Exeter, New Hampshire

Southeast—William A. Gager, University of Florida, Gainesville, Florida

North Central—Irene Sauble, Detroit Public Schools, Detroit, Michigan

Northwest—Elizabeth Roudebush, 815-4th Avenue North, Seattle 9, Washington

Southwest—Ida May Bernhard, Texas Education Agency, Austin 11, Texas

Miss Mary C. Rogers, Chairman of the

Committee on Affiliated Groups and Vice-president of the National Council, appointed the above committee and has placed in its hands the responsibility for the forthcoming assembly.

PROGRAM—8:00 TO 11:00 A.M.

Part I

Opening Keynote
Mary C. Rogers, Chairman of the Committee on Affiliated Groups
John R. Mayor, President, NCTM

The new Affiliated Group Handbook
Dr. H. Glenn Ayre, Chairman

(Discussion following)

A Forward Look at Our Future NCTM Yearbooks
Dr. Francis Lankford, Chairman

(Discussion following)

Future Plans for our National Journals
Dr. Henry Van Engen, Editor of THE MATHEMATICS TEACHER
Dr. Ben A. Sueltz, Editor of *The Arithmetic Teacher*
Dr. Harold D. Larsen, Editor of the *Mathematics Student Journal*

(Discussion following)

The Affiliated Group Newsletter
Dr. H. Glenn Ayre, Editor

(Discussion following)

Brief announcements by other committee chairmen.

INTERMISSION

Part II

Effective Affiliated Group Programs
Discussion led by panel of delegates and supplemented by open discussion of techniques used to improve programs.

General discussion centering on the following topics:

(1) How can we improve liaison between the field, the Council officers, and the national office at the N.E.A.?
(2) How can publicity concerning activities of the Affiliated Groups be improved.
(3) What additional responsibility for National Council affairs should be delegated to the Affiliated groups? What responsibilities should the Delegate Assembly have for the State Representatives of the Council.
(4) In what specific ways can the Affiliated Groups assist in raising the professional standards and training of mathematics teachers?

DINNER PROGRAM—5:30 TO 7:30 P.M.

An interesting program and a minimum of speeches are to be planned. You are invited (requested, urged, exhorted) to communicate with us, making suggestions for a unique feature on this evening program. We, naturally, prefer a mathematical program, but we want something that you feel will go over, or that you have used successfully in your group.

Furthermore, we need volunteers for the panel mentioned above. If you can't volunteer, write us anyway, giving us a helpful idea or even just a cheerio! After all, this is YOUR meeting. It will be a good one if you help make it so. And so together we say, "On to Cincinnati!"

"Solon was under a delusion when he said that a man when he grows old may learn many things—for he can no more learn much than he can run much; youth is the time for any extraordinary toil."

—PLATO, *The Republic*

"Round numbers are always false."

—SAMUEL JOHNSON

"To unlearn the taken-for-granted is harder than to learn the hitherto unsuspected."

—IRVING FISHER

Your 1954 Convention City

by Mildred Keiffer, Cincinnati Public Schools, Cincinnati, Ohio

Cincinnati welcomes you to the Thirty-second Annual Meeting of the National Council of Teachers of Mathematics. Built upon a circle of hills around a wide, flat basin, Cincinnati is one of America's most picturesque big cities, offering panoramic views from hill-top parks and streets of the winding Ohio River Valley and Kentucky beyond.

Cincinnati Union Terminal, one of the most impressive and functional depots in the world, boasts a whispering gallery not so famous as St. Paul's in London or the Capitol's in Washington but surpassing both for distance the voice is carried. Visitors will want to see the Terrace Plaza, an outstanding example of ultra-modern architecture; and quaint Mt. Adams, one of the oldest sections of the city, with streets and steps descending at odd angles. Formerly called Mt. Ida, its name was changed after John Quincy Adams laid the cornerstone of the observatory there. Mostly vineyards in pioneer days, it is now the home of Rookwood Pottery and the Church of the Immaculate Conception to which hundreds make a pilgrimage on Good Friday.

Taft Museum, once the Longworth home, designed for Cincinnati's first mayor by the architect who restored the White House after 1814, is within walking distance of the headquarters hotel, the Sheraton-Gibson. The Cincinnati Art Museum invites all visitors to see its nationally recognized collections. The Clifton Observatory memorializes Dr. Cleveland Abbe, father of the U. S. Weather Bureau. St. Francis de Sales Church is famous for its $17\frac{1}{2}$-ton bell housed in a 230-foot steeple and said to be the largest swinging bell in the United States. It rang only once, in January, 1896 and has not been rung since to spare the nearby buildings and windows.

The first Saengerfest of America was held by Cincinnati Germans in 1849 and was the forerunner and inspiration of the Cincinnati May Festival, biennial since 1873 a year after the organization of the Cincinnati Symphony Orchestra.

Industrially speaking, Cincinnati holds a high position with its many diversified interests. In the production of soap, machine tools, playing cards, jet engines, and electrotypes, it ranks second to none. It is the only city in the world which owns its own railroad.

The Tyler-Davidson Memorial Fountain, brought from Munich and erected on Cincinnati's Fountain Square in 1871, can be seen from the north windows of the Sheraton-Gibson Hotel. Standing on the highest pedestal is the "Genius of Water," while below four figures dramatize the allegory of water.

The many attractive residential areas of Cincinnati will be beautiful in the spring with blossoming shrubs and trees. The Sight-Seeing Committee is making plans to show all these attractions and many more, to the delegates who attend the April 21–24 meeting of the National Council of Teachers of Mathematics in Cincinnati.

Report of the Nominating Committee

The Committee on Nominations and Elections presents the persons listed below as candidates for officers and directors of the Council with two candidates for each position. The term for President and the Vice Presidents is two years, and the term for a Director is three years. The Committee wishes to thank the many members

who proposed the names of more than a hundred people for the various offices, a tribute to the alertness of the members and to the efficient leadership of workers in many parts of the country. The committee regrets that recognition by nomination cannot be given to all of these able individuals whose names were submitted.

Election ballots will be mailed on March 1, 1954 to all persons who are members on that date. Ballots will also be mailed to persons whose membership orders are postmarked not later than March 1, 1954. Ballots should be marked and returned promptly and must be postmarked not later than April 1, 1954 to be valid.

MARY A. POTTER, *Chairman*
IDA MAY BERNHARD
HARRY CHARLESWORTH
MARTHA HILDEBRANDT
LESTA HOEL
VIRGIL MALLORY
W. W. RANKIN
ROLLAND R. SMITH
BEN A. SUELTZ
HENRY VAN ENGEN

NOMINEES FOR PRESIDENT

MARIE S. WILCOX

AGNES HERBERT

Marie S. Wilcox

WILCOX, MARIE S., Thomas Carr Howe High School, Indianapolis, Indiana.

A.B. and M.A. in Mathematics, Indiana University.

Teacher of Mathematics; Brown Township High School, Indiana; New Albany High School, New Albany, Indiana; George Washington High School, Indianapolis, Indiana; Head of Mathematics Department, Thomas Carr Howe High School, Indianapolis, Indiana; Butler University Summer Session, 1945. Study Group Leader: Mathematics Conference, The University of Wisconsin, 1951; Louisiana State University Mathematics Institute, 1952 and 1953.

Member: NCTM; CAMST (Board of Directors, 1935–40, President, 1939); Mathematics Section, Indiana State Teachers' Association (Vice-President, 1946, President, 1947); Indiana University Women's Club (President 1937–38); Indianapolis Panhellenic Association (President, 1930); Phi Mu Fraternity (District President, 1940–43; National Scholarship Director, 1943–48; National Alumnae Vice-President, 1948–50; National Extension Director, 1950–52); NEA; Phi Beta Kappa; Mortar Board; Pi Lambda Theta; Indianapolis Council of Administrative Women in Education. Listed in *Leaders in Education* and *Who's Who in American Education.*

Activities in NCTM include Member of Board of Directors, 1948–51; Vice-President, 1952–54; Chairman of Committee on Revisions of By-Laws, 1950–51; Chairman of Committee on Selection of Editor for High School Publication, 1953; Program Chairman, 1953 Christmas meeting. Member of committees: Executive, 1952–54; Budget, 1950–52; Selection of Editor for THE MATHEMATICS TEACHER, 1950 and 1953; Contests and Scholarships, 1948–53; Program for Pittsburgh, Stillwater, Atlantic City, Kalamazoo, and Cincinnati meetings.

Agnes Herbert

HERBERT, AGNES, Clifton Park Junior High School, Baltimore, Maryland.

Graduate of Baltimore City Teachers Training College, now Towson State Teachers College, Towson, Maryland; graduate of Teachers College, Columbia University; additional study at Johns Hopkins University; University of Maryland; New York University.

Teacher in elementary school, two years; teacher of junior high school mathematics and Chairman of the Department of Mathematics, Clifton Park Junior High School; demonstration teacher at Johns Hopkins University Summer School, 1947; participated in the United States Armed Forces Institute program at Fort George G. Meade, 1943–46.

Member of NCTM; Overseas Teacher Relief Committee for Maryland State Teachers, 1948–49 (Chairman, 1949–50); NEA. (member of Resolutions Committee for Maryland 1950–51 and 1951–52); Mathematics Section of the Maryland State Teachers Association (one of organizers, Treasurer for six years, Chairman for two years).

Activities in the NCTM include member of Board of Directors, 1949–52; state representative; Chairman for Twenty-seventh Annual Meeting of NCTM in Baltimore in 1949; appearance on its convention programs; Vice-President in 1952–53, Recording Secretary 1953–54, Workshop Programs in Arlington, Virginia, Marshall College, W. Virginia.

NOMINEES FOR VICE-PRESIDENT—ELEMENTARY LEVEL

E. GLENADINE GIBB

CHARLOTTE W. JUNGE

E. Glenadine Gibb

GIBB, E. GLENADINE, Assistant Professor of Mathematics, Iowa State Teachers College, Cedar Falls, Iowa.

B.Ed., Western Illinois State College; M.A., George Peabody College for Teachers; Ph.D., University of Wisconsin.

Instructor in mathematics, H.S., Mendon, Illinois, 1941–1945; H.S., Geneseo, Illinois, 1945–1946; Iowa State Teachers College, 1946–; Director, Workshops in The Teaching of Arithmetic, Marshall College, Summers, 1950, 1951; Visiting Lecturer in Education and teacher in 5th–6th grade in University Laboratory School, University of Wisconsin, Summer, 1951.

Member of NCTM (Iowa delegate to Delegate Assembly, 1950, member of local arrangements committee, Madison and Des Moines, membership committee); CASMT (Chairman of Elementary Mathematics Section, 1953, vice-chairman, 1952, secretary, 1951; member of Board of Directors, 1951–1954; Policy and Resolutions Committee, 1953–1955); NEA, Iowa Association of Mathematics Teachers (President, 1950, vice-president, 1949); Iowa State Education Association, AAUP; AAUW; American Statistical Association; Delta Kappa Gamma; Pi Lambda Theta; Kappa Mu Epsilon.

Contributions to "What Is Going On In Your School?" and "Book Section," THE MATHEMATICS TEACHER; "A Review of a Decade of Research Which Compared Methods of Teaching Arithmetic," *Journal of Educational Research.*

Charlotte W. Junge

JUNGE, CHARLOTTE W., College of Education, Wayne University, Detroit, Michigan.

B.A., Colorado State College of Education; M.A., Ph.D., State University of Iowa.

Teacher, District 10, Sedwick Co., Colorado, 1929–1931; Merino, Colorado, 1931–1935. Primary Supervisor, Big Bend Training School, Colorado State College of Education, 1936–1940. Teacher, University Elementary School, The University of Iowa, 1940–1944 (Acting

Principal 1942–1943). Principal, College Elementary School, and Professor of Education, State Teachers College, Moorhead, Minnesota, 1944–1946. Associate Professor, College of Education, Wayne University, 1946–.

Summer Sessions: 1945, 1946, and 1947, visiting professor at Cornell University, Ithaca, New York; 1950, visiting professor, Northwestern University; 1953, visiting professor, The University of Illinois. Nature of duties: Teaching and research in Elementary Education with emphasis on the teaching of Arithmetic; duties involve working with Senior College, Master's and Doctor's students.

Member of Board of Directors of the Central Association of Science and Mathematics, 1952–1954; President, Michigan Association for Student Teaching, 1949–1951; Member NEA, MEA, NCTM, CASMT, NSSE, Michigan Association for Student Teaching, National Association for Student Teaching.

Co-author of *Growth in Arithmetic*, a series of textbooks for elementary grades; articles in *School Science and Mathematics*, *NEA Journal* and ACE bulletins. Co-author of *Bibliography in Elementary Education*, published in 1950 and again in 1952 by the ASCD; Co-author of chapter in NSSE Fiftieth Yearbook, Part II, 1951; author of chapter in 1950 Yearbook of the Association for Student Teaching.

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B.A., State University of Iowa; M.S., Northwestern University; Ph.D., State University of Iowa.

Activities in NCTM include: Member of Commission on Post-War Plans, 1945–1947; Chairman of Banquet Committee, 30th Annual Convention, 1952; Secretary of Symposium on Teacher Education in Mathematics, University of Wisconsin, 1952; Member of Committee on Publications of Current Interest; Member of Committee on Revision of the Guidance Pamphlet.

Teacher of Mathematics, Thornton Fractional Township High School, Calumet City, Illinois, 1931–1934; Teacher of Mathematics, University High School, Iowa City, Iowa, 1934– (Head of Department and Supervisor of Teacher-Training since 1940); Department of Mathematics, State University of Iowa, 1940–; Supervisor of ASTP, Basic Phase, army training program in mathematics at the State University of Iowa, 1942–1943; Visiting Professor, School of Education, University of Michigan, Summer 1952.

Member, Mathematical Association of America, Central Association of Science and Mathematics Teachers (Board of Directors, Policy and Resolutions Committee, Sectional officer), Iowa Association of Mathematics Teachers (various offices), Chairman of State Curriculum Revision Committee in Mathematics, Phi Beta Kappa, Sigma Xi, Phi Delta Kappa. Listed in *American Men of Science*.

Co-author of *Basic Skills in Mathematics*. Author of a variety of examinations for Iowa and Missouri state-wide testing programs, Adjutant General's Office, and Cooperative Test Service; "Mathematics Clubs" and "The Small High School" in THE MATHEMATICS TEACHER; "We *Can* Remove the Stigma from General Mathematics" and "An Experiment in Fusing Plane and Solid Geometry" in *School Science and Mathematics*.

Jackson B. Adkins

ADKINS, JACKSON B., The Phillips Exeter Academy, Exeter, New Hampshire.

Ph.B., University of Chicago; Ed.M., Harvard University.

Teacher of Mathematics, Central Junior High School, Lima, Ohio, 1923–25; Bloom Township High School, Chicago Heights, Illinois, 1926–27; Proviso Township High School, Maywood, Illinois, 1927–29; Culver Military Academy, Culver, Indiana, 1929–32; Moses Brown School, Providence, Rhode Island, 1933–39; Phillips Exeter Academy, Exeter, New Hampshire, 1939–. Teacher, U. S. Navy Midshipman School, New York, 1942–44; Head of Mathematics Department, U. S. Navy Premidshipman School, Asbury Park and Princeton, New Jersey, 1944–45.

Member of NCTM; (Member of Committee on the Official Journal, Committee on Affiliated Groups). Member Am. Math. Assn., Member of Association of Teachers of Mathematics in New England (Board of Directors, 1947–50, '51–'53, Speakers Bureau, 1951–52, Vice-president, 1952–); Program Chairman, New England Institute for Teachers of Mathematics, 1951; General Chairman of the Joint Meeting of the Institute and the NCTM, 1952; Chairman of Local Arrangements for the spring meeting of the NCTM in Boston, 1955. Founder and President of Southeastern New Hampshire Association of Teachers of Mathematics, 1946–. New Hampshire State Committee on the Revision of the Senior High School Curriculum in Mathematics. Co-author, *A First Course in Algebra*. Phi Delta Kappa; Tau Kappa Epsilon.

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BELL, CLIFFORD, Professor of Mathematics, and Head of Mathematics Extension, Los Angeles Campus, University of California.

B.S. and Ph.D., University of California, Berkeley.

Activities in NCTM include: Membership Committee; Institute Committee; Joint Committee on Cooperation with Industry and the

later Committee on Cooperation with Industry; Chairman, Local Arrangements Committee, Fourteenth Christmas Meeting in Los Angeles.

Teaching Fellow University of California, Berkeley 1923–25; Instructor to Professor, University of California, Los Angeles 1925–; Educational Supervisor in Charge of Mathematics Courses, Southern California area, University of California ESMWT program 1942–45. In charge of teacher-training courses in Department of Mathematics, University of California, Los Angeles.

Member of NCTM, California Council of Teachers of Mathematics, MAA, American Mathematical Society, AAAS, Society for the Advancement of Education, and a number of honor societies.

Chairman of the Planning Committee of the Annual California Conference for Teachers of Mathematics and Director of the 1953 Conference.

Co-author of textbooks in trigonometry, commercial algebra, and the mathematics of finance. Author of articles in the *Bulletin of the American Mathematical Society*, Monthly of the MAA, THE MATHEMATICS TEACHER, and other national and foreign journals.

Listed in *American Men of Science*, *Who's Who in Education* and other biographical books.

Emil J. Berger

BERGER, EMIL J., Instructor in Mathematics and Chairman of the Department, Monroe High School, St. Paul, Minnesota; also Part-time Instructor in Mathematics, College of St. Catherine, St. Paul, Minnesota.

B.E., State Teachers College, St. Cloud, Minnesota; M.A., Colorado State College of Education, Greeley, Colorado; further graduate study, University of Minnesota.

Instructor in Mathematics, Kimball, Minnesota (1939–41); High School Principal, Remer, Minnesota (1941–42); Instructor in Mathematics and Science, Dowling College High School, Des Moines, Iowa (1942–43); U.S. Army, Transportation Corps and Public Relations Section, NYPE (1943–45); Instructor in Mathematics, Monroe High School (1946–) and College of St. Catherine (1947–); Guest Lecturer, College of Education, University of Minnesota (Summer, 1951).

Member of NCTM; MAA; Minnesota Council of Teachers of Mathematics (Board of Directors, 1949–; Vice President, 1952–53; President, 1953–); Minnesota Educational Association (Mathematics Section President, 1953–). Has served as a speaker for neighboring state and local meetings of mathematics teachers, and was a member of the Minnesota State Curriculum Committee for Mathematics (1950–53). Associate editor of *Minnesota Mathematics News Letter* (1951–).

Activities in the NCTM include appearances on its convention programs; presentation of exhibits at conventions (School Exhibits Chairman, Summer Meeting, 1951); representative to the Delegate Assembly (1950); program committee (Annual Meeting, 1952); contributor to *Twenty-Second Yearbook of NCTM;* Editor of the department, "Devices for a Mathematics Laboratory," THE MATHEMATICS TEACHER.

Mary Lee Foster

FOSTER, MARY LEE, Henderson State Teachers College, Arkadelphia, Arkansas.

B.A., M.S., Louisiana State University. Additional graduate study: University of Alabama; Louisiana State University; Teachers College, Columbia University.

Teaching experience in high schools: Amite, Louisiana; Alexandria, Louisiana; Greenwood, Mississippi. Teaching fellowships: University of Alabama, Louisiana State University. Scholarship: Outstanding Public School Teacher, Louisiana State University. Instructor of Mathematics, Henderson State Teachers College, 1937–44; Head of Department of Mathematics, Huntingdon College, Montgomery, Alabama, 1944–45; Adjunct Professor of Mathematics, University of South Carolina, 1945–46; Associate Professor of Mathematics, Henderson State Teachers College, 1946–.

Member: AAUW, American Math. Assn.; Pi Mu Epsilon. State representative NCTM, 1949–. President of Mathematics Section of Arkansas Education Assn., 1951–. Publication: "The Ford Experiment in Arkansas," THE MATHEMATICS TEACHER, April 1953.

William A. Gager

GAGER, WILLIAM A., Professor of Mathematics, University of Florida, Gainesville, Florida.

B.S., M.S., Pennsylvania State College; Ph.D., Peabody College for Teachers.

Instructor in Engineering, Pennsylvania State College, 1919–1923; teacher of mathematics and science, St. Petersburg High School, 1927–42; summer teaching, Peabody College for Teachers, 1936–39; Associate Professor of Mathematics, University of Florida, 1942–49; Professor, 1949–54. Mathematics consultant, Florida Department of Education, 1940, 1942, 1948, 1949, 1950; chairman, Florida Curriculum Commission, 1947–50; chairman of Mathematics Clinic for State Supervisors Conference, 1948; director of State Department Study of Mathematics in Secondary Schools of Florida, 1947–49; editor-in-chief, Duke *Mathematics at Work*, 1948–52; editor and director, Florida State Bulletins No. 36 and No. 40; director of Florida Council of Teachers of Mathematics, 1950; consultant on Education for the Talented, U.S. Office of Education, 1952.

Activities in NCTM include: associate editor, THE MATHEMATICS TEACHER, 1950–53; board member, 1951–54; sectional director of delegate group, 1952–54; Membership Committee, 1952; secretary of Symposium for

Preparation of Mathematics Teachers, 1952; editor of newsletter, 1952.

Author: *Terminology of Business Mathematics in Junior College; Mathematics for War Effort; Functional Mathematics, Grades 9 & 10,* 1953; "Proper Appreciation of Mathematics," FEA 1935; "Computing with Approximate Numbers," SSM 1947; "Mathematics for the Other Eighty-Five Per cent," SSM 1948; "Aiding Students in Acquiring Basic Concepts," DHL 1949; "A Functional Program for Secondary Mathematics," THE MATHEMATICS TEACHER 1949; "Quality Control in Secondary Mathematics," DHL 1950; "Concepts for Certain Mathematics Courses," SSM 1950; "Functional Mathematics, Grades Seven Through Twelve," THE MATHEMATICS TEACHER 1951; "The Changing Mathematics Program," NEA 1951; "Newton and His *Principia Mathematica,*" SSM 1952.

Member: NCTM, CASMT, MAA, Florida Academy of Science, Florida Education Association, Florida Public Health Association, AAUP, Phi Kappa Phi, Tau Beta Pi, Pi Delta Kappa, Kappa Delta Pi. Active participant in local church, community, and fraternal activities.

Houston T. Karnes

KARNES, HOUSTON T., Professor of Mathematics, Louisiana State University, Baton Rouge, Louisiana.

A.B., A.M., Vanderbilt University; Ph.D., Peabody College for Teachers; summer sessions, University of Wisconsin and the University of Michigan.

Activities in NCTM include: Louisiana-Mississippi State Representative; Member of Planning Commission of 1952 Symposium on Teacher Education in Mathematics; Co-editor of the Department, "What Is Going on in Your School?" of THE MATHEMATICS TEACHER; served on several committees of NCTM and MAA.

Professor of Mathematics and Biology, Northwestern Junior College, Orange City, Iowa, 1929–35; Professor of Mathematics and Dean of Men, Harding College, Searcy, Arkansas, 1935–36; Teacher of Mathematics and Department Head, high schools, Nashville, Tennessee, 1936–38; Assistant in Mathematics, Louisiana State University, 1938–40, Instructor, 1940–42, Assistant Professor, 1942–45, Associate Professor, 1945–53, Professor, 1953– . Dean of Men, Louisiana State University during the war years. In the summers, visiting professor in the College of Education, Louisiana State University, conducting courses in teacher training. Member of MAA; American Mathematical Society; National Education Association; Louisiana Education Association; American Association of University Professors; Pi Mu Epsilon (Associate Editor of Pi Mu Epsilon Journal); Kappa Mu Epsilon; Phi Delta Kappa; Kappa Delta Pi; Omicron Delta Kappa; Board of Trustees, Harding College; Louisiana-Mississippi Branch of National Council of Teachers of Mathematics (Past chairman, Recorder, Delegate Assembly); Director of the Louisiana State University Mathematics Institute; Chairman, Standing Committee of Mathematicians of Louisiana for the Furtherance of Mathematical Education. Past secretary and past chairman of the Mathematics Section of the Louisiana College Conference. Past secretary of the Louisiana College Conference. Past president of the Louisiana State University Faculty Club. International president of the Lambda Chi Alpha Fraternity. Read many papers before annual meetings of the National Council of Teachers of Mathematics. Listed in *American Men of Science, Who's Who in American Education* and the *Supplement of Who Knows and What.* Author of "Preparation of Teachers of Secondary Mathematics" and "Junior College Mathematics Curriculum Problems in View of the President's Report," THE MATHEMATICS TEACHER; and "Legislation vs. Education," *American Mathematical Monthly.* Filling a one year unexpired term on the Board of Directors of the NCTM.

Catherine A. V. Lyons

LYONS, CATHERINE A. V., University School, Pittsburgh, Pennsylvania.

B.A., M.A., Ph.D., University of Pittsburgh. Attended Universities: Harvard, Wisconsin, Chicago, Washington, and U.C.L.A.

All teaching experience in Pittsburgh. Taught Mathematics from third grade through twelfth grade. Taught four summers in the Demonstration School formerly at the University of Pittsburgh, and four years in the Veteran's Training Program. Now teaching senior high school mathematics in a private school.

Member of NCTM since 1930.

Former officer and present executive committee member of Western Pennsylvania Association of Teachers of Mathematics and State Round Table in Mathematics; Organizing Chairman and present President of Pennsylvania Council of Teachers of Mathematics, 1950– . Vice President and Program Chairman of AAUW; Secretary of the Doctoral Association of the University of Pittsburgh. On board and committees: Pi Lambda Theta; Business and Professional Women; other professional organizations. Co-chairman of NCTM Meeting in Pittsburgh in 1951.

Has worked on a number of Mathematics and Reading Committees (Curriculum and Revision), including that for the new Pennsylvania State Bulletin No. 360. Has published short articles in local bulletins, and newsletters.

Announcement of Program for Fellowships for High School Teachers

The Fund for the Advancement of Education is herewith announcing a program of fellowships for approximately 300 public secondary school teachers throughout the United States and its territories for the academic year 1954–55. This program is designed to permit the recipients to devote a year away from the classroom to activities that will extend their liberal education, improve their teaching ability, and increase their effectiveness as members of their school systems and communities.

It is the Fund's expectation that such an opportunity afforded to teachers of demonstrated ability will make a substantial contribution to the improvement of secondary teaching throughout this country.

The responsibility for designing the year's program rests primarily upon the candidate. Because this fellowship program is concerned primarily with the broadening of the individual, it is *not* designed to include those types of specialized activity in which the teacher has traditionally engaged during the summer months or during previous years away from the classroom, such as taking additional courses toward a graduate degree in a major subject or field in which the teacher has already had extensive training, or courses for credit in professional education.

In short, the teacher should plan the most stimulating year that he can conceive in behalf of his personal enrichment as a teacher.

The National Committee on High School Teacher Fellowships now believes that *all* teachers in public secondary school systems should have an opportunity to compete for these fellowships. At the same time, it recognizes that local school officials and citizens are best qualified to nominate teachers who can benefit most from this program. Accordingly, the superintendent of any secondary school district where one or more teachers desire to make application under this announcement is requested to appoint a local committee to nominate the most appropriate candidates from that district.

All classroom teachers in junior and senior high schools who have the necessary qualifications may enter the local competition. Eligibility for a fellowship is limited to teachers (1) who have taught at least three years and have devoted at least half time to classroom teaching in each of the past three academic years, and (2) who will not be more than fifty years of age on April 15, 1954.

Forms for both individual applicants and for local nominating committees are being distributed to superintendents in all high school districts throughout the country. A limited number of additional forms may be obtained from the National Committee on High School Teacher Fellowships, 575 Madison Avenue, New York 22, New York.

Individual applicants should not apply to the Fund for the Advancement of Education but only to their superintendent of schools or local nominating committee.

The amount of the fellowship award will be generally equivalent to the regular salary the teacher would receive during the school year (excluding summer, night school, or other "extra" work), but no less than $3,000, plus reasonable allotments for necessary transportation expenses or for tuition in case the teacher registers at an institution for additional work. Only costs of transportation within the continental limits of the United States may be covered by the grant, though a fellowship recipient is free to undertake foreign travel at his own expense.

The recommendations of the local committees should be mailed so as to reach the offices of the National Committee on High School Teacher Fellowships no later than March 1, 1954. Final announcement of all fellowship awards will be made on or about April 15, 1954.

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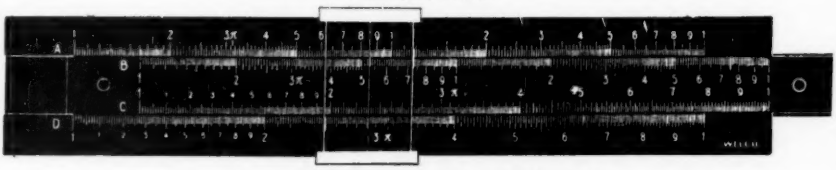

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